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ANALYSIS AND DESIGN OF AUTOMATIC
GAIN CONTROL SYSTEMS

by

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Expressions are derived for the loop gain, and its influence in the regulation characteristics of the loop is shown.

The frequency response of AGC loops is predicted and used to reshape transient and steady state response making use of conventional frequency response diagrams.

Using the derived expressions the design of AGC loops is quite straightforward and their characteristics such as regulation and effect on distortion are readily predictable.

A non-linear gain characteristic was derived generating no distortion (under certain assumptions), and the analytical expression of it was derived.

The developed theory and design techniques were proved to be applicable to AGC loops with arbitrary number of dynamics in the forward as well as in the feedback path.

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Analysis and Design of Automatic
Gain Control Systems

by

John Paloubis
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Submitted in partial fulfillment of the
requirements for the degree of

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I. INTRODUCTION

Automatic Gain Control (AGC) is a closed loop regulating system the purpose of which is to provide closely controlled signal amplitude at the output, despite the variation of amplitude and frequency in the input signal. The above goal is generally accomplished by feeding back a measure of the output signal and through this adjusting the gain by which the input signal is multiplied.

Historically the first appearance of AGC systems in electronics can be placed as early as 1923 when a form of AGC was obtained by using triodes, biased from the detector, to shunt the antenna circuit. In this instance it was intended for limiting the noise produced by strong atmospherics. A later method employed a mechanical control to reduce the capacitance between the antenna and receiver; the moving coil of a milliammeter connected in the detector anode circuit actuated the moving vanes of the antenna capacitance.

The introduction of variable mu R.F. tubes marked a most important step in the history of AGC, for control of R.F. gain by grid bias became possible. The bias was derived from the D.C. component of the detected carrier output voltage.

Today with the immense expansion of electronic's applications, AGC systems are used almost inevitably in electronic

equipment like RADARS, TV, COMMUNICATION RECEIVERS, COMPUTERS, etc.

Although the discussion so far is restricted to the field of electronics, AGC systems constitute a broader category including a variety of problems.

Wherever a constant nominal output level has to be maintained, despite the input variations, a kind of AGC system can be employed; so problems of constant pressure, temperature, fluid or gas flow, etc. can be examined, analyzed and designed as AGC problems.

There exists a large amount of literature about AGC systems treating in various ways the normal AGC system as it is encountered in communications, TV or RADAR's receivers, the main characteristic of which is that it has a low pass filter or an ideal integrator in the feedback path.

Only special cases of AGC loops have been solved completely in an analytical way and as the order of the system gets higher than one, there doesn't seem to be any way of solving the non-linear differential equation relating the input to the output.

Even for first order systems one has to make certain assumptions as far as the nature of the gain of the voltage controlled amplifier is concerned in order to be able to proceed analytically.

References [1] and [2] contain practical considerations of instrumentation of an AVC loop for a regular superheterodyne receiver and explain in a practical way the

effect of the reference signal on the AGC action. Similar information can be found in any receiver handbook.

Reference [3] treats the single-pole AGC system as a feedback control problem and calculates the reference signal from the static requirements of the loop.

Reference [4] without theoretical development describes an AGC system applied successfully to microwaves which produced an attenuation of input fluctuations of the order of 40 db.

Reference [5] introduces a combination of a sync clipper and an AGC circuit used in TV receivers in a highly effective noise gating arrangement, discriminating against impulse noise.

Reference [6] describes an AGC system for the usual transistorized superheterodyne receivers and obtaining AGC control from a single grounded base stage succeeds in saving a diode.

Reference [7] remains as an almost classic treatment of AGC systems. The authors arbitrarily chose the non-linear gain as being an exponential function of the bias voltage (result at which this thesis arrives in Chapter IV considering the static requirements around the loop seeking a distortionless characteristic when both input and output are expressed in decibels with respect to unity).

Using Wiener methods they designate the closed loop transfer function of an AGC system which gives the minimum rms error, in receiver gain, and which turns out to be a

single-pole low pass filter and which yields the feedback filter of the system as being an ideal integrator.

Reference [8] presents an alternative method of derivation of the same results as reference [7] plus some additional details of performance.

Reference [9] treats an AGC loop containing a non-linear feedback filter.

Reference [10] analyzes an AGC loop with a single pole in the feedback path and proceeding further proves analytically that with a double-pole filter it is possible to excite a regenerative mode in the system and the transients become unbounded.

Reference [11] analyzes a regular AVC system in the presence of stationary Gaussian noise.

The purpose of this thesis is to investigate AGC systems with higher dynamics and the associated problems with them. Such systems are encountered as temperature regulators, fluid or gas flow regulators and generally wherever time delays in the transmission of signals or actual components create an arbitrary number of poles and zeros around the loop.

It is intended to derive approximate formulas which will enable one to predict and analyze the behavior of these systems to a certain extent, and will provide a good basis for practical design and compensation of AGC loops.

From this point of view an AGC system can be thought of as a self-adaptive servomechanism which controls the gain of an amplifier by feeding back a measure of the output in

such a way that the nominal amplitude level of the output signal remains constant independent of the amplitude variations of the input signal.

There is a distinction between the "regulator" type of systems and the usual AVC systems encountered in common receivers. The former try to keep the output amplitude constant in all cases. The latter try to keep the nominal output amplitude constant but it is undesirable to suppress the overriding high frequency signal which carry the information to be received. In this thesis an attempt is made to include both of them under the same broad category of Automatic Gain Control systems leaving the distinction to be taken care of by the location of the input variation frequency with respect to the frequency band contained in the bandwidth of the individual systems.

Specifically it is proved that if the input variation frequency is located outside the bandwidth of the AGC loop the variations are suppressed by the AGC action, whereas if the input variation frequency lies inside the passband of the loop the AGC action has no effect on the input variations which pass undisturbed.

Conventional feedback control theory is used throughout this work since there is a great resemblence between AGC loops and feedback control loops. This permits the well established techniques of feedback control to be applied to the AGC problem.

The feedback nature of a typical AGC system becomes clearer considering the essential features of a simple

feedback amplifier.

1. There is an input.
2. There is an output.
3. There is a transmission path which develops a measure of the output.
4. There is a means for comparing this measure of the output with a reference level, i.e. means for developing an "error" signal which is the algebraic sum of the reference and the measure of the output.
5. There is an amplifier which uses this "error" signal to adjust the output.

Thus it is seen that the description of a simple feedback amplifier differs in only a few details from the description of a typical AGC system.

II. FORMULATION OF AGC PROBLEM

In Fig. 1 an AGC system is shown in block schematic form which consists of:

1. An amplifier whose gain $N(v)$ depends on the amplitude of a control signal.
2. An arbitrary number of poles and zeros in $G_1(S)$, $G_2(S)$, $G_3(S)$ depending on the physical time constants of the actual system.
3. A constant reference or threshold signal V_{REF}.
4. The input signal V_{in}.

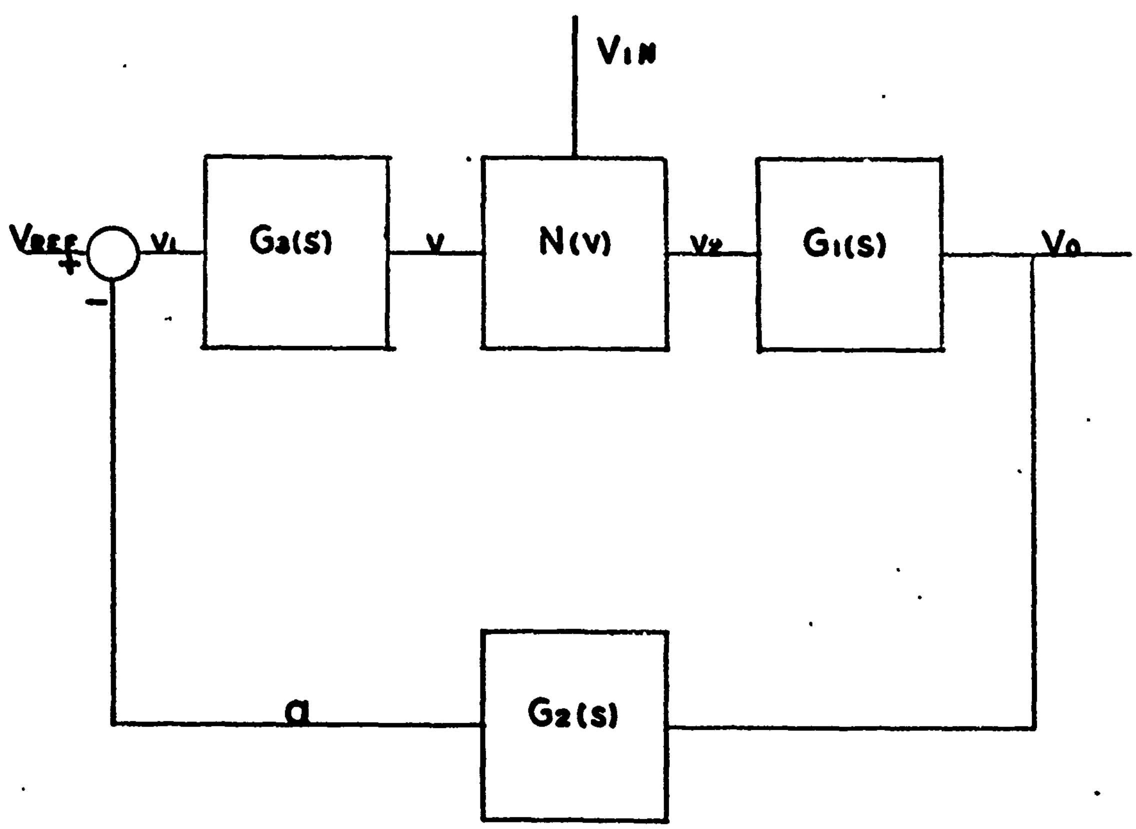


Figure 1. Block Diagram Representation of a Typical AGC System.

5. The output signal Vo (the amplitude of which must be kept constant).

If $G1(S) = 1$, $G3(S) = 1$ and $G2(S)$ consists of a low pass filter the usual AGC system of communication receivers appears, the purpose of which is to keep the nominal value of the envelope constant.

If Vin is the temperature of incoming gas and $N(\nu)$ represents a chamber in which the temperature is raised or lowered in order for the output temperature to be constant, a temperature regulating system is considered.

If V_{REF} is replaced by an audio frequency input then a "radio transmitter with envelope feedback" is considered, and then the amplifier constitutes a modulator.

So a great variety of systems can be calssified under the broad category of AGC systems.

The AGC action can be more clearly understood if the system is considered as a conventional feedback loop having as input V_{REF} and as output Vo. In this case Vin is considered as a signal source in the loop and the voltage controlled amplifier as a non-linear block with input ν and output $Vin \cdot N(\nu)$.

Assuming the transmission gain in the forward path as "μ" and in the feedback path as "β" the following equations can be easily derived.

$$Vo = \mu(V_{REF} - \beta Vo) \qquad (2\text{-}1)$$

or

$$Vo = \frac{\mu}{1+\mu\beta} V_{REF} \tag{2-2}$$

and if $\mu\beta \gg 1$

$$Vo \approx \frac{1}{\beta} V_{REF} \tag{2-3}$$

and the output is independent of "μ," so that disturbances in the forward path are suppressed. The degree of the suppression may be obtained by differentiating (2-2) with respect to "μ" and dividing the result by (2-2) itself.

Thus

$$dVo = \frac{d\mu}{(1+\mu\beta)^2} \cdot V_{REF} \tag{2-4}$$

and

$$\frac{dVo}{Vo} = \frac{1}{1+\mu\beta} \quad \frac{d\mu}{\mu} \tag{2-5}$$

So any variations in the forward path of the loop are suppressed by the factor $\frac{1}{1+\mu\beta}$.

In the temperature regulating system, and the communications receiver's AVC, these variations are fluctuations of the external temperature, or received rf signal strength correspondingly. So long as $|\mu\beta| \gg 1$ these variations will be suppressed, and a nominal output amplitude Vo equal to $\frac{1}{\beta} V_{REF}$ (and therefore constant) will be developed.

In the case of AVC $|\mu\beta|$ must be less than unity over the range of desired modulation frequencies in Vin or these modulations also would be suppressed in the output.

In a radio transmitter the principal "μ" - circuit variations might be the non-linearity of the modulator, and

$|\mu\beta|$ would be made much larger than unity over the entire modulation frequency spectrum.

Equation (2-5) indicates the importance of V_{REF}. If V_{REF} were zero, any output which might appear would be due to the failure of the circuit to regulate completely.

In many AVC systems (called "undelayed" systems) the reference voltage is zero.

That this type of system performs satisfactorily in many applications arises from the fact that the loop gain is low for small received-signal amplitudes as will be shown in Chapter III. The "failure to regulate" is thus quite large for low signal inputs and the regulation does not become "good" until an appreciable (and usable) output has been developed.

Thus, in conclusion from the feedback point of view an AGC system is a D.C. amplifier-modulator with negative-envelope feedback, whose input is a constant voltage and whose average output amplitude therefore is constant so long as the loop gain is high.

III. ANALYSIS OF AGC LOOPS

A. LINEARIZATION-LOOP GAIN

Before an approximate solution of AGC systems is attempted an exact solution of a simple loop is presented for the purpose of showing the difficulty of an analytical approach even for the simplest of the cases.

Assume the block diagram of Figure 2, where everything else is 1 except $G3(S) = \frac{1}{S}$ an ideal integrator.

Suppose also $N(\nu)$ is given by $N(\nu)=e^{a\nu}$ (which is a very desirable function for non-linear gain).

Tracing the signals around the loop the following is derived as the differential equation of the system.

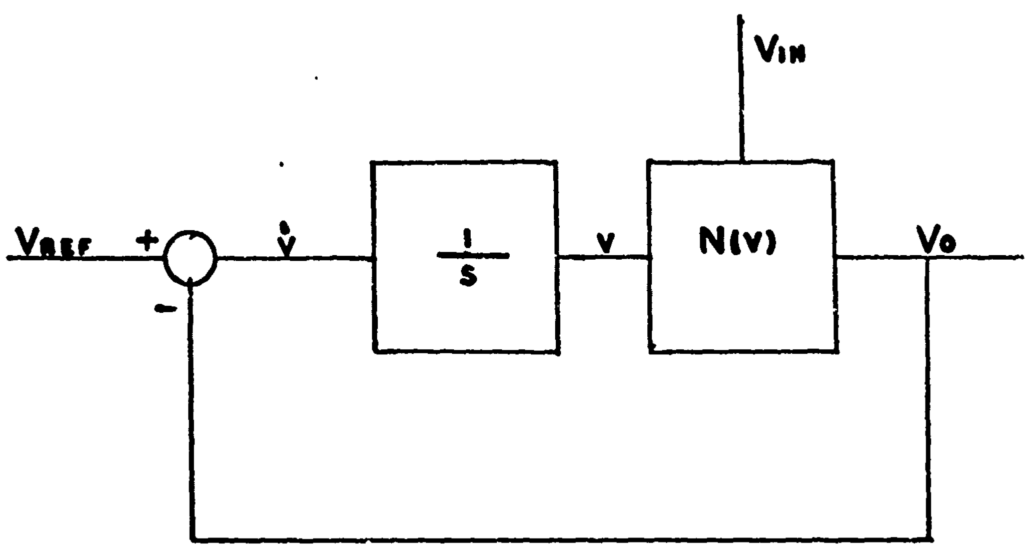


Figure 2. Block Diagram of a Simplified AGC Loop.

$$\dot{v} = V_{REF} - V_{in}(t)\, N(v) \tag{3-1}$$

and by substituting the gain expression

$$\dot{v} = V_{REF} - V_{in}(t)e^{av} \tag{3-2}$$

which is a first order non-linear differential equation. This equation is difficult to solve even for simple input waveforms. The simple but important case of a step input will be investigated. Specifically it will be assumed that the input:

$$V_{in}(t) = \overline{V_{in}} \tag{3-3}$$

has been applied for a long period of time and steady-state has been reached. At time $t = t_1$ (where t_1 can be taken as zero) a step change in the input occurs

$$V_{in}(t) = M \qquad\qquad t > 0 \tag{3-4}$$

In the period $t < 0$ where steady-state has been reached equation (3-2) becomes

$$0 = V_{REF} - \overline{V_{in}}\, e^{av} \tag{3-5}$$

and solving for v

$$v = \frac{1}{a} \ln \left(\frac{V_{REF}}{\overline{V_{in}}}\right) \tag{3-6}$$

which in the initial value of v when the step change in $V_{in}(t)$ occurs, i.e.

$$v_o = \frac{1}{a} \ln \left(\frac{V_{REF}}{\overline{V_{in}}}\right) \quad \text{at} \quad t = 0 \tag{3-7}$$

At $t = 0_+$ equation (3-2) becomes:

$$\dot{v} = V_{REF} - Me^{av} \tag{3-8}$$

or

$$\frac{dv}{V_{REF}-Me^{av}} = dt \tag{3-9}$$

and integrating

$$\int \frac{dv}{V_{REF}-Me^{av}} = \int dt + C \tag{3-10}$$

The left-hand side integral was found evaluated in reference [12] p. 92 #2.313-1 and results in

$$\frac{1}{aV_{REF}}\left[av - \ln(V_{REF}-Me^{av})\right] = t + C \tag{3-11}$$

Setting t = 0, v has the value given by equation(3-7) and then substituting

$$C = \frac{1}{aV_{REF}}\left[\ln\left(\frac{V_{REF}}{\overline{V}_{in}}\right) - \ln\left(V_{REF} - \frac{M}{\overline{V}_{in}} V_{REF}\right)\right]$$

$$= \frac{1}{aV_{REF}} \ln\left[\frac{\frac{V_{REF}}{\overline{V}_{in}}}{V_{REF}\left(1 - \frac{M}{\overline{V}_{in}}\right)}\right] = \frac{1}{aV_{REF}} \ln\left(\frac{1}{\overline{V}_{in}-M}\right) \tag{3-12}$$

Substituting (3-12) into (3-11) results in:

$$\frac{1}{aV_{REF}}\left[av - \ln(V_{REF}-Me^{av})\right] = t + \frac{1}{aV_{REF}} \ln\left(\frac{1}{\overline{V}_{in}-M}\right) \tag{3-13}$$

Simplifying more,

$$av - \ln\left(V_{REF} - Me^{av}\right) = aV_{REF}t + \ln\left(\frac{1}{\overline{V_{in}} - M}\right) \qquad (3\text{-}14)$$

or

$$av - \ln\left(\frac{V_{REF} - Me^{av}}{\overline{V_{in}} - M}\right) = aV_{REF}t \qquad (3\text{-}15)$$

Now letting $t \to \infty$ one can get the final value of v which is

$$v_f = \frac{1}{a} \ln\left(\frac{V_{REF}}{M}\right) \qquad (3\text{-}16)$$

and agrees with equation (3-7) for $V_{in}(t) = \overline{V_{in}}$.

Substituting some values for a, V_{in}, V_{REF}, M it can be seen that the plot of v versus time will be as sketched in Figure 3.

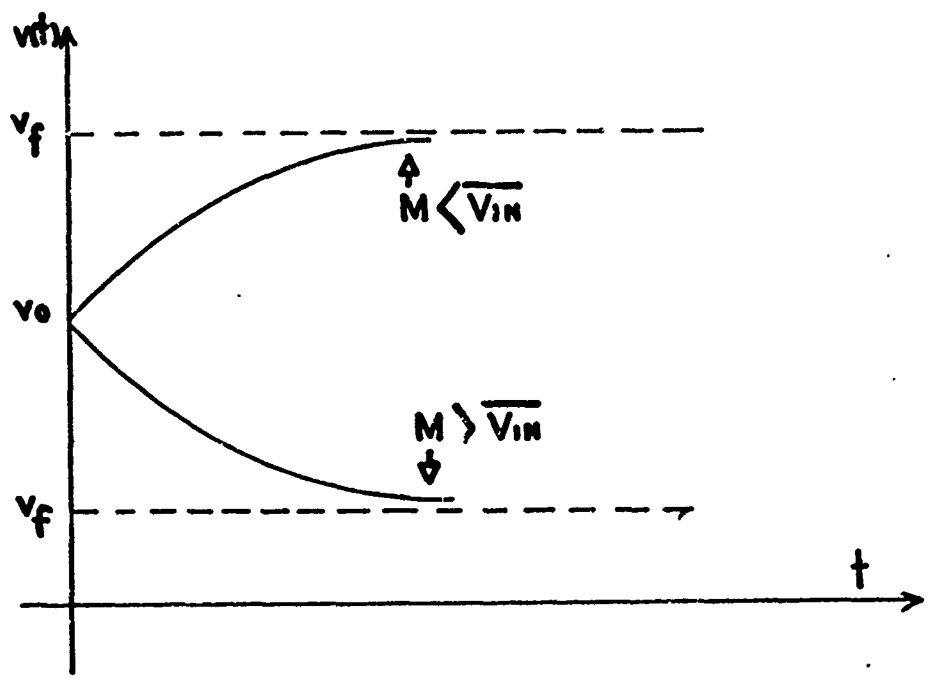


Figure 3. Sketch of v Versus Time.

Correspondingly the gain $N(v) = e^{av}$ will have similar variations with respect to v, i.e. increasing v, N(v) will increase and vice versa.

The output of the system $Vo(t) = V_{in}(t)N(v)$ plotted against time will have the characteristics shown in the sketch of Figure 4.

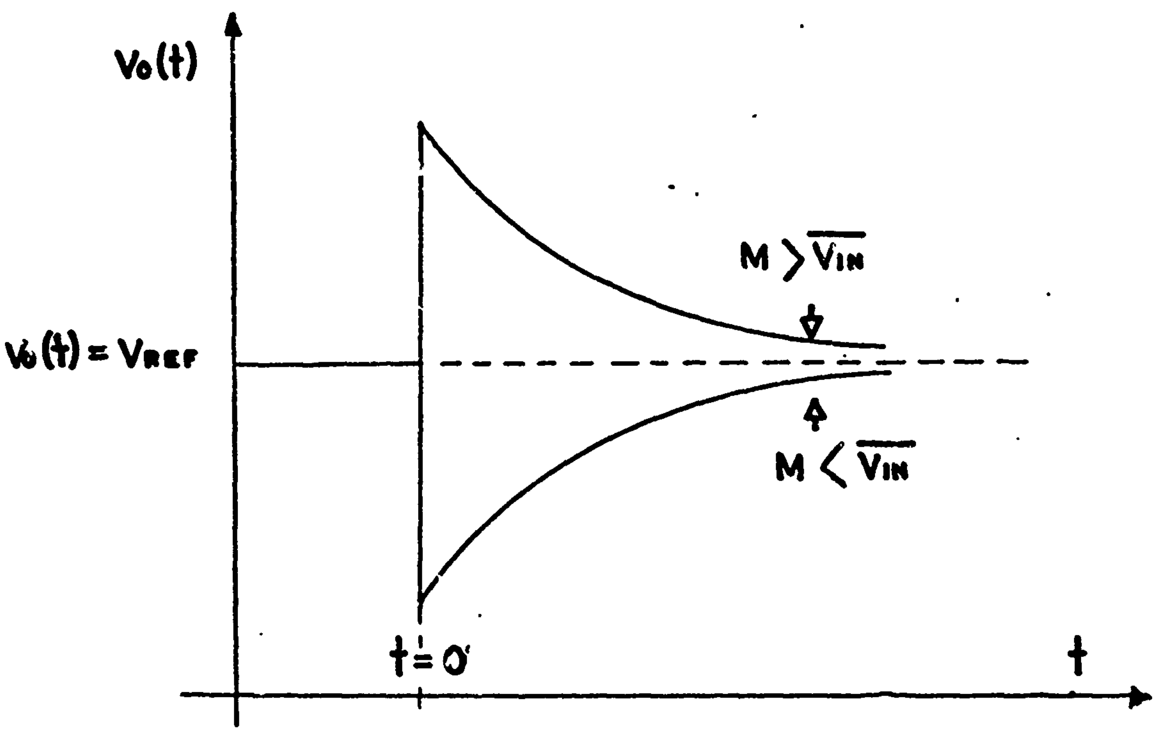


Figure 4. Sketch of Vo(t) Versus Time.

The analytically solved example was merely intended to show that even the simpler AGC loop results in an almost impossible to solve differential equation if the input waveform $V_{in}(t)$ is other than a simple constant value.

The following is an approximate analysis and it is based on small signal operation of a generalized AGC system

thus permitting linearization of the loop around an equilibrium point.

It will be assumed that the input $V_{in}(t)$ has a constant value and steady state has been established in the operation of the system. Then small perturbations at the input will be related to resulting perturbations at the output.

The assumption can be expressed mathematically as:

$$V_{in}(t) = \overline{V_{in}} + \delta V_{in}(t) \tag{3-17}$$

and

$$Vo(t) = \overline{V}o + \delta Vo(t) \tag{3-18}$$

where $\overline{V_{in}}$ is a constant amplitude input resulting in a constant amplitude output $\overline{V}o$ and $\delta V_{in}(t)$ is a small signal perturbation superimposed on $\overline{V_{in}}$ and resulting in small amplitude fluctuation at the output $\delta Vo(t)$.

From the assumption it also follows that the non-linear gain:

$$N(v) = Ao + A_1 v + A_2 v^2 + \ldots \tag{3-19}$$

can be linearized around the operating region and can be considered as:

$$N(v) = A_0 + A_1 v \tag{3-20}$$

Referring back to Figure 1 and remembering that $v(t) = \overline{v} + \delta v(t)$ the following equations are derived.

$$v_2(t) = \bar{v}_2+\delta v_2(t) = V_{in}(t)\cdot N(v) = (\overline{V_{in}}+\delta V_{in}(t))(A_o+A_1 v)$$

$$= (\overline{V_{in}}+\delta V_{in}(t))(A_o+A_1\bar{v}+A_1\delta v(t))$$

$$= (A_o+A_1\bar{v})\overline{V_{in}} + (A_o+A_1\bar{v})\delta V_{in}(t) + \overline{V_{in}}A_1\delta v(t)$$

$$+ A_1\delta V_{in}(t)\delta v(t). \tag{3-21}$$

where the barred quantities denote the nominal values caused by the constant amplitude input $\overline{V_{in}}$ and the differential quantities (δ) are those resulting from the differential input perturbation $\delta V_{in}(t)$.

In the above equation the last term is the product of two differentials and can be neglected compared with the others, resulting in

$$\bar{v}_2 + \delta v_2(t) = (A_o+A_1\bar{v})\overline{V_{in}} + (A_o+A_1\bar{v})\delta V_{in}(t) + A_1\overline{V_{in}}\delta v(t) \tag{3-22}$$

Letting the perturbation $\delta V_{in}(t) = 0$ and consequently $\delta v(t) = 0$, $\delta v_2(t) = 0$

$$\bar{v}_2 = (A_o + A_1\bar{v})\ \overline{V_{in}} \tag{3-23}$$

which is an expected result.

Subtraction of eqn. (3-23) from (3-22) gives

$$\delta v_2(t) = (A_o+A_1\bar{v})\delta V_{in}(t) + A_1\overline{V_{in}}\ \delta v(t) \tag{3-24}$$

and taking the Laplace transform

$$\delta v_2(s) = (A_o+A_1\bar{v})\delta V_{in}(s) + A_1\overline{V_{in}}\ \delta v(s) \tag{3-25}$$

Now tracing the signals around the loop the following equations can be derived:

$$\delta V_o(s) = G_1(s)\delta v_2(s) \tag{3-26}$$

$$\delta a(s) = G_2(s)\delta V_o(s) \tag{3-27}$$

$$\delta v_1(s) = -\ \delta a(s) \tag{3-28}$$

$$\delta\ v(s) = G_3(s)\delta v_1(s) \tag{3-29}$$

Manipulating equations (3-29), (3-28), (3-27)

$$\begin{aligned}\delta v(s) &= G_3(s)\delta v_1(s) = -G_3(s)\delta a(s)\\ &= -G_3(s)G_2(s)\delta V_o(s)\end{aligned} \tag{3-30}$$

Substituting this equation into (3-25)

$$\delta v_2(s) = (A_o+A_1\bar{v})\delta V_{in}(s) - A_1\overline{V_{in}}G_2(s)G_3(s)\delta V_o(s) \tag{3-31}$$

Substituting the last equation into (3-26)

$$\delta V_o(s) = (A_o+A_1\bar{v})\delta V_{in}(s)G_1(s) - A_1\overline{V_{in}}G_1(s)G_2(s)G_3(s)\delta V_o(s) \tag{3-32}$$

and rearranging:

$$\delta V_o(s)[1+ A_1\overline{V_{in}}G_1(s)G_2(s)G_3(s)] = (A_o+A_1\bar{v})G_1(s)\delta V_{in}(s) \tag{3-33}$$

or

$$\frac{\delta V_o(s)}{\delta V_{in}(s)} = \frac{(A_o+A_1\bar{v})\ G_1(s)}{1 + A_1\overline{V_{in}}G_1(s)G_2(s)G_3(s)} \tag{3-34}$$

which is the transfer function of the linearized loop around an operating point.

The same result could be obtained using a state variable approach as follows:

Suppose the block of Figure 1 denoted by its transfer function $G_1(s)$ is characterized by

$$\dot{x}^{(1)} = A_1 x^{(1)} + b_1 v_2 \tag{3-35}$$

and the output equation

$$V_o(t) = C_1^T x^{(1)} + d_1 v_2 \tag{3-36}$$

Applying linearization procedure assume

$$x^{(1)} = x_e^{(1)} + \delta x^{(1)} \tag{3-37}$$

where $x_e^{(1)}$ is the equilibrium value of the vector $x^{(1)}$ and $\delta x^{(1)}$ is the perturbation vector.

Differentiating (3-37) with respect to time

$$\dot{x}^{(1)} = \dot{x}_e^{(1)} + \delta\dot{x}^{(1)} = A_1(x_e^{(1)} + \delta x^{(1)}) + b_1(\bar{v}_2 + \delta v_2)$$

$$= A_1 x_e^{(1)} + b_1\bar{v}_2 + A_1\delta x^{(1)} + b_1\delta v_2 \tag{3-38}$$

But $\dot{x}_e^{(1)} = 0$ because $x_e^{(1)}$ = constant and therefore $A_1 x^{(1)} + b_1\delta\bar{v}_2 = 0$ and remains

$$\dot{x}^{(1)} = A_1\delta x^{(1)} + b_1\delta v_2 \tag{3-39}$$

In the output equation .

$$V_o(t) = \bar{V}_o + \delta V_o(t) = C_1^T\ (x_e^{(1)} + x^{(1)}) + d_1(\bar{v}_2 + v_2)$$

$$= C_1^T x_e^{(1)} + d_1\bar{v}_2 + C_1^T\delta x^{(1)} + d_1\delta v_2 \tag{3-40}$$

and from this it turns out that

$$\delta V_o(t) = C_1^T\delta x^{(1)} + d_1\delta v_2 \tag{3-41}$$

Equations (3-39) and (3-41) represent the state and output equations of the linearized relations between input and output of block one. In a similar way the following are easily derived:

$$\text{block 2}\begin{cases}\delta \dot{x}^{(2)} = A_2 \delta x^{(2)} + b_2 \delta V_o & (3\text{-}42)\\ \delta a = C_2^T \delta x^{(2)} + d_2 \delta V_o & (3\text{-}43)\end{cases}$$

$$\text{block 3}\begin{cases}\delta \dot{x}^{(3)} = A_3 \delta x^{(3)} + b_3 \delta v_1 & (3\text{-}44)\\ \delta v = C_3^T \delta x^{(3)} + d_3 \delta v_1 & (3\text{-}45)\end{cases}$$

$$\delta v_1 = -\delta a \qquad (3\text{-}46)$$

$$\delta v_2 = (A_o + A_1 \bar{v}) \delta V_{in} + A_1 \overline{V_{in}} \delta v \qquad (3\text{-}47)$$

The last two equations are exactly derived as equations (3- !8) and (3-25). Now taking the Laplace transform of Eqn. (3-39) and solving

$$\delta x^{(1)}(s) = (sI - A_1)^{-1} b_1 \delta v_2(s) \qquad (3\text{-}48)$$

Substituting into the transformed Eqn. (3-41)

$$\delta V_o(s) = C_1^T \delta x^{(1)}(s) + d_1 \delta v_2(s) = C_1^T (sI - A_1)^{-1} b_1 \delta v_2(s)$$

$$+ d_1 \delta v_2(s) = [C_1^T (sI - A_1)^{-1} b_1 + d_1] \delta v_2(s) \qquad (3\text{-}49)$$

But $\delta v_2(s)$ is given transforming equation (3-47)

$$\delta v_2(s) = (A_o + A_1 \bar{v}) \delta V_{in}(s) + A_1 \overline{V_{in}} \delta v(s) \qquad (3\text{-}50)$$

and substituting into Eqn. (3-49)

$$\delta V_o(s) = [C_1^T (sI - A_1)^{-1} b_1 + d_1] \cdot [(A_o + A_1 \bar{v}) \delta V_{in}(s) + A_1 \overline{V_{in}} \delta v(s)]$$

$$= [C_1^T (sI - A_1)^{-1} b_1 + d_1] (A_o + A_1 \bar{v}) \delta V_{in}(s)$$

$$+ [C_1^T (sI - A_1)^{-1} b_1 + d_1] A_1 \overline{V_{in}} \delta v(s) \qquad (3\text{-}51)$$

In a similar fashion transforming equations (3-45), (3-44), (3-43), (3-42) and (3-46)

$$
\begin{aligned}
\delta v(s) &= C_3^T \delta x^{(3)}(s) + d_3 \delta v_1(s) \\
&= [C_3^T (sI-A_3)^{-1} \cdot b_3 + d_3]\ \delta v_1(s) \\
&= -\ [C_3^T (sI-A_3)^{-1} b_3 + d_3]\ \delta a(s) \\
&= -\ [C_3^T (sI-A_3)^{-1}\ b_3 + d_3][C_2^T \delta x^{(2)}(s) + d_2 \delta V_o(s)] \\
&= -\ [C_3^T (sI-A_3)^{-1} b_3 + d_3][C_2^T (sI-A_2)^{-1} b_2 \delta V_o(s) + d_2 \delta V_o(s)] \\
&= -\ [C_3^T (sI-A_3)^{-1} b_3 + d_3][C_2^T (sI-A_2)^{-1} b_2 + d_2] \delta V_o(s)
\end{aligned}
\tag{3-52}
$$

and substituting into Eqn. (3-51)

$$
\begin{aligned}
\delta V_o(s) = {} & [C_1^T (sI-A_1)^{-1} b_1 + d_1](A_o + A_1 \bar{v}) \delta V_{in}(s) \\
& -A_1 \overline{V_{in}} \delta V_o(s) [C_1^T (sI-A_1)^{-1} b_1 + d_1][C_2^T (sI-A_2)^{-1} b_2 + d_2][C_3^T (sI-A_3)^{-1} b_3 + d_3]
\end{aligned}
\tag{3-53}
$$

If in this last expression the factors in brackets are recognized as the transfer functions of the individual blocks we conclude with the same expression as the small signal transfer function.

The denominator of the RHS of equation (3-34) is the characteristic equation of the linearized loop

$$1 + A_1 \overline{V_{in}} G_1(s) G_2(s) G_3(s) = 0 \tag{3-54}$$

It is readily observed that the loop gain of the AGC system is the product of three terms: the nominal input amplitude, times the slope of the non-linear gain characteristic at the

operating region, times whatever linear gain exists inthe three block transfer functions $G_1(s)$, $G_2(s)$, $G_3(s)$.

At this point in order to derive a much more useful relationship between the loop gain and the regulation that can be obtained by the system, assume the restricted but widely applicable case, in which the three block transfer functions are of type zero and consider

$$G_j(0) = 1 \qquad j = 1,2,3 \tag{3-55}$$

and associated with them the linear gains

$$G_j \qquad j = 1,2,3 \tag{3-56}$$

In this case the loop gain can be expressed as

$$(\text{Loop gain}) = G_1 G_2 G_3 \cdot \overline{V_{in}} \; \frac{dN(v)}{dv} \tag{3-57}$$

Since

$$\overline{V_{in}} = \frac{\bar{V}_2}{N(v)} = \frac{\bar{V}_0}{G_1 N(v)} \tag{3-58}$$

Substituting into Eqn. (3-57)

$$\begin{aligned}(\text{Loop gain}) &= G_2 G_3 \, \bar{V}_o \, \frac{1}{N(v)} \, \frac{dN(v)}{dv} \\ &= G_2 G_3 \, \bar{V}_o \, \frac{d(\ln N(v))}{dv} \\ &= G_2 G_3 \, \bar{V}_o \, \frac{d}{dv} \left[\frac{\log_{10} N(v)}{\log_{10} e} \right] \\ &= G_2 G_3 \, \bar{V}_o \, \frac{1}{20 \log_{10} e} \, \frac{d}{dv} \, [20 \log_{10} N(v)] \\ &= 0.115 \, G_2 G_3 \, \bar{V}_o \, \frac{d}{dv} \, [20 \log_{10} N(v)] \end{aligned} \tag{3-59}$$

In this expression it is observed that use has been made of the slope of the gain characteristic at the operating region when the gain is expressed in decibels with respect to unity.

Since the general assumption of small signal analysis holds, it can be assumed that the characteristic of the gain when expressed in db versus bias voltage v is linear and it is given by

$$20 \log_{10}[N(v)] = Sv + C \qquad (3\text{-}60)$$

where s is the slope in db per AGC bias volt and C is a constant. Such a linear characteristic when the gain is expressed in db is depicted in Fig. 5.

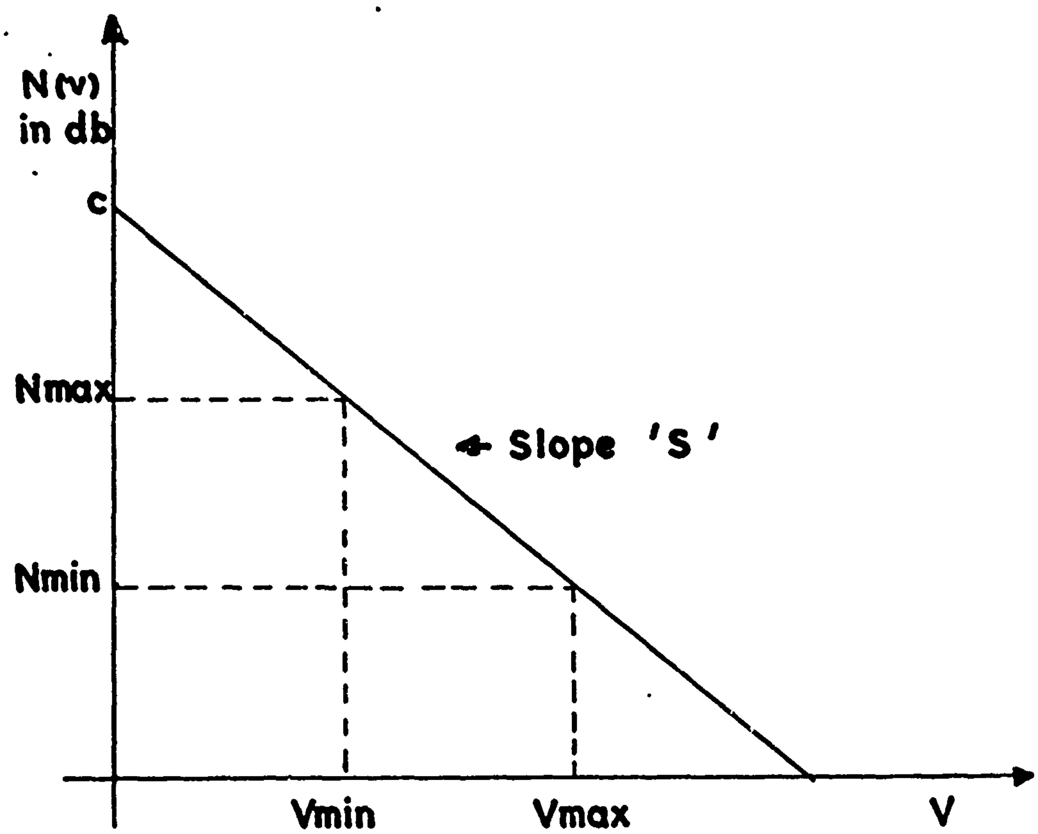


Figure 5. Linear Amplifier Gain Control Characteristic.

From the picture it is seen that

$$S = \frac{Nmax - Nmin}{Vmax - Vmin} \tag{3-61}$$

Substituting this last expression into equation (3-59)

$$\text{Loop gain} = \frac{0.115\ G_2G_3\ \overline{V}_o\ [Nmax-Nmin]}{Vmax - Vmin} \tag{3-62}$$

At this point it is observed that the actual change in the gain is the amount of regulation that can be obtained by the system and also considering the transmission of the signals around the loop.

$$\frac{Vmax - Vmin}{G_2} = V_o max - V_o min \tag{3-63}$$

Substituting into Eqn. (3-62)

$$\text{Loop gain} = \frac{0.115\ G_3\ \overline{V}_o\ [\text{Gain change in db}]}{V_o max - V_o min} \tag{3-64}$$

It is apparent from this expression that with the linear gain control characteristic shown in Fig. 5 the loop gain will vary somewhat with the nominal output amplitude $\overline{V}_o$ but the output amplitude will be normally well enough controlled that its variations can be neglected and an average value used in equation (3-64).

To illustrate the use of equation (3-64) suppose that static input variations of 100 db must be regulated by the AGC loop to output variations of 2 db (± 1 db around the nominal value of V_o). This means that V_o must vary between 0.89 $\overline{V}_o$ and 1.122 $\overline{V}_o$. Suppose also $G_3 = 1$. Substituting the numbers into equation (3-64) yields

$$\text{Loop gain} = \frac{(0.115)(100)}{1.122-0.89} = 49.5 = 33.4 \text{ db}$$

Thus with an AGC loop gain of about 50, input variations of 100 db can be suppressed to output variations of only $\pm$ 1 db.

This example is supposed to illustrate how the formula is applied in a practical case and may not work in practice because input variations of 100 db may wipe out the linearization assumption.

The conclusion so far is that the regulation achieved by an AGC system depends upon the loop gain and this depends on the linear gain of the system, the input amplitude and the slope of the amplifier gain characteristic.

In Appendix "A" an example has been worked which verifies equation (3-57). In this example an actual AGC loop has been simulated repeatedly, each time keeping constant two of the factors contained in the loop gain expression (linear gain, nominal value of the input, slope of the characteristic) and varying the third.

Since the regulation depends on the loop gain, various degrees of regulation observed, prove equation (3-57).

B. STABILITY - TRANSIENT RESPONSE

The significance of the derived characteristic equation (3-54) is that the majority of the techniques of analysis and design of linear feedback loops are applicable to the case of AGC systems taking into consideration the nonlinearity of the gain factor.

For example the stability of the system can be determined using the root-locus plot, as follows:

Knowing the slope of the operating region of the non-linear gain plot, and the linear gain of the loop, the loop-gain depends entirely and linearly on the amplitude of the input signal. Then substituting the maximum expected input amplitude, the maximum possible loop gain can be determined. If for this maximum possible loop gain the roots remain in the left-hand plane of the root-locus plot then clearly the system is stable. If on the other hand the roots are driven into the right-hand plane the system becomes unstable for a part of its operation. If for the lower expected input amplitude the loop-gain is higher than the critical value which can be determined by Routh's criterion then the system is completely unstable.

The behavior of the system in case of instability can be forseen arguing on the physical operation of the system, and it turns out that it is very similar to that of a linear feedback loop containing a saturated amplifier. Thus if the AGC loop is unstable the output amplitude tends to grow and consequently the non-linear gain drops in an effort to keep the output voltage in the prescribed bounds.

Then the gain gets too low and the output amplitude gets smaller than it should be resulting in a tendency of increasing gain.

Thus a limit cycle is established, the frequency of which can be predicted by the root-locus of the system, (the fre-

quency at which the root locus crosses the imaginary axis), and the amplitude depends on how far in the right-hand plane are the roots of the system (i.e. it depends on the loop-gain).

The location of the AGC roots on the s-plane determines also the characteristics of the transient response. However, since this location changes continuously with the input amplitude, the features of the transient response are not always the same.

There exist, of course, systems like the previously mentioned temperature regulating loop, where the variations of the input amplitude are expected to be very slow compared with the time constants involved in the system and then, to a good approximation, the loop can be considered always working in the steady state and the transient response is not so much of interest.

This however does not obscure the importance of the transient response characteristics which is very critical in other applications. The only thing that can be predicted for the transient response is that the various features remain between easily predicted limits. Since the root location is what specifies the transient response, a region on the s-plane can be determined in which the roots move depending upon the input amplitude. If the range of the input amplitude is not large then the region becomes smaller and the characteristics of the transient response obtain more closely spaced limits.

To obtain well specified transient response one should try to keep the loop gain as constant as possible. But the loop gain as it was derived previously is given by:

(Loop gain) = (Linear gain) x (Input amplitude)
x (Slope of characteristic)

The linear gain of the loop after it is set cannot be changed, so the slope of the characteristic only can provide the required compensation to the input amplitude fluctuations. This means that the slope of the characteristic should not remain constant during the operation but should change in an opposite sense to the input to keep the loop gain as constant as possible.

So, from this point of view a curvature in the non-linear gain characteristic is desirable.

A typical example of desired gain characteristic is given in Fig. 6.

The basic operation of the loop employing this characteristic is as follows. If the input amplitude increases the output amplitude and correspondingly the bias voltage increases absolutely and the operating slope of the characteristic decreases keeping the loop gain reasonably constant.

It is well known from linear feedback theory that the question of stability arises if and only if there exist more than three poles in the system.

In the usual AVC systems employed in communications receivers the stability of the loop is not considered at all since the loop usually contains only one pole in the feedback path.

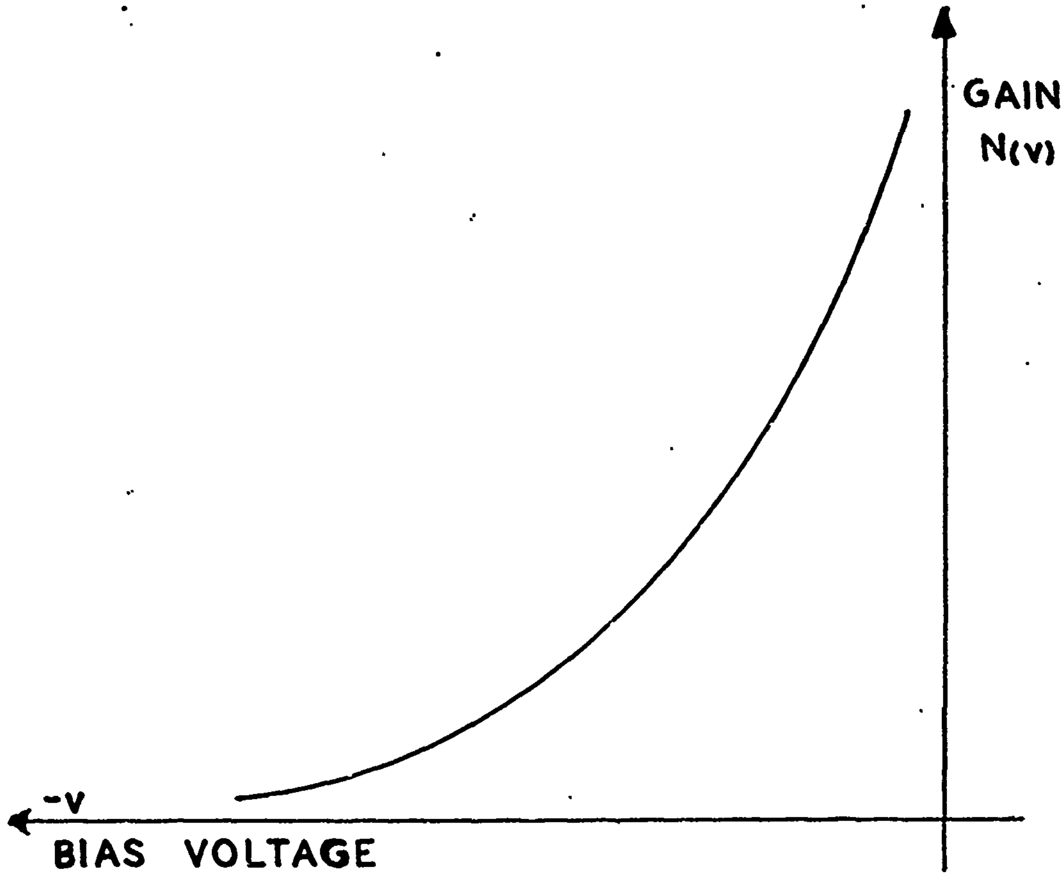


Figure.6. Typical Example of Desired Gain Characteristic.

In Appendix "B" an example has been worked which verifies that an unstable AGC loop ends up in a limit cycle. In this example an AGC loop containing three poles has been simulated. From the conventional BODE plot the exact value of $\overline{V_{in}}$ for which the system becomes unstable has been predicted and verified by the simulation.

C. BIAS - DETERMINATION OF REFERENCE

The following treatment is based on static regulation requirements of the AGC system and its purpose is to derive expressions relating the various quantities around the loop.

The following action will ordinarily be desired.

1. If the input amplitude is so weak that with the maximum gain of the non-linear amplifier times the linear gain G_1 the output is less than a desired minimum value no gain reduction should be produced by the AGC.

2. If the input amplitude has the maximum expected value, the output amplitude should not exceed a certain permissible value, and with this value of output the AGC circuit must produce the required gain reduction in the non-linear amplifier.

Let

Vomin = minimum desired value of Vo

Vomax = maximum permissible value of Vo

vmin = control voltage required to produce maximum gain

vmax = control voltage required to produce minimum gain

then condition (1) requires that, for Vo = Vomin v = vmin, and then

$$\bar{v} = (V_{ref} - G_2\overline{V}_o)G_3 \tag{3-65}$$

Substituting the values derived from condition (1) and solving for V_{REF} the value of the reference voltage is de rived:

$$V_{REF} = \frac{G_2G_3Vomin + vmin}{G_3} \tag{3-66}$$

If the potential required to produce maximum gain is zero as is usually the case then the value of the reference

voltage becomes:

$$V_{REF} = G_2 Vomin \tag{3-67}$$

Evaluating equation (3-65) for Vomax:

$$vmax = (V_{REF} - G_2 Vomax)G_3 \tag{3-68}$$

and substituting (3-66 and solving for the product G_2G_3:

$$G_2G_3 = \frac{vmin - vmax}{Vomax - Vomin} \tag{3-69}$$

where it is assumed that the bias voltage v is negative.

If vmin is zero

$$G_2G_3 = \frac{- vmax}{Vomax - Vomin} \tag{3-70}$$

Thus equations (3-66) and (3-69) give the necessary reference voltage and amplification in the feedback path correspondingly in order to meet the static regulation requirements.

Some thoughts about the relationships interconnecting the various quantities around the AGC loop may now be expressed.

Obviously the interpretation of equation (3-69) is that, a trial is made to "match" the output amplitude range into the working range of the bias voltage. As the range of the bias voltage increases, keeping the output amplitude range constant, the linear gain G_2G_3 increases and consequently the loop gain tends to increase. But increasing the bias voltage range means that the slope of the non-linear gain

characteristic decreases, counter balancing the effect of the linear gain and so keeping the loop gain constant. The constancy of the loop gain means unchaged regulation of the output amplitude, which was the condition derived in the previous section.

Another important characteristic of the AGC loop is that in the absence of Vin there exists no output and consequently the control voltage becomes zero raising the non-linear gain to its maximum value. Therefore the next time an input signal is applied the non-linear gain is maximum and another transient response takes place until the steady-state operation is again established.

This kind of operation is especially bothersome when the input consists of an intermittent waveform or bursts of data.

It is then clear that before a steady-state operation has been established, the input signal vanishes driving the non-linear gain to its maximum value and when the input signal reappears a new transient starts again.

It would be desirable to maintain as long as possible the same gain during the silence periods of the input signal, which means to maintain the control voltage "v" almost constant.

This is partly accomplished employing two time constants in the feedback path either in $G_2(s)$ or in $G_3(s)$. Figure 7 shows a schematic representation of a L.P. filter having two different time constants, which can be employed as described above.

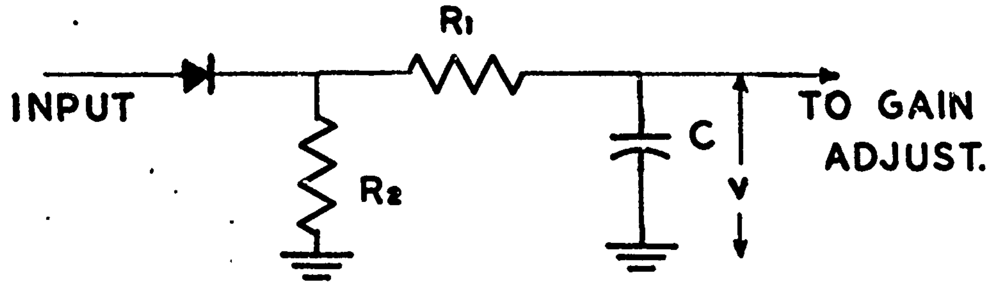


Figure 7. L.P. Filter with Two Time Constants.

If the potential across the capacitor C is the control voltage "v" which determines the value of the non-linear gain then the charge time constant of the capacitor is R_1C. and this can be designed to meet the other requirement of the specific problem. If the input signal vanishes then the discharge time constant of the capacitor becomes $(R_1+R_2)C$ which can be very large depending upon the resistor R_2.

Therefore the voltage "v" remains almost constant and consequently the next burst of data which comes meets the same gain as before, reassuming steady-state operation very quickly.

However, care should be taken in using the above circuit or any other similar to this. If the design is not good the AGC loop becomes insensitive to rapid decreases of input level and then the main purpose of the AGC loop, (to keep constant nominal output amplitude) is not accomplished.

The time constant $(R_1+R_2)C$ must be calculated taking into account the expected rate of input amplitude variations.

In Appendix C a simulation was attempted of an AGC loop showing the effect of the reference value on the regulation of the output. It is essentially shown there that AGC action starts when the output value exceeds the value Vomin = $\frac{V_{REF}}{G_2}$ derived from the equation (3-67) and does not occur when the output amplitude is lower than this value.

D. FREQUENCY RESPONSE

Some methods have already been applied in the frequency domain in order to assert the stability of the AGC loop when it contains more than 2 poles. (Interpretation and verification of the prediction of BODE plot, see Appendix B.)

In terms of conventional control theory the AGC loop is a non-unity feedback system and in order to draw the various plots (BODE, ROOT LOCUS) use has been made of the open loop transfer function GH. That is, the product of the transfer functions of all the filters around the loop has been considered or in other words the derived characteristic equation (3-54) was plotted. As was pointed out previously the non-linear factor the "Loop gain" appearing in equation (3-54) does not change appreciably the theory on which is based the interpretation of BODE and ROOT-LOCUS plots. In simple terms, as gain changes during the operation of the loop, the roots of the system move on the ROOT-LOCUS or the magnitude curve of the BODE plot moves vertically thus changing the gain and phase margins.

Since these quantities do not remain constant during the operation of the loop one is not able to predict operational characteristics of the system as peak overshoot, settling time, bandwidth, etc. which are estimated for conventional loops given the root location or the phase and/or the gain margin. Not only prediction is not possible, but one also cannot attach-meaning to these quantities in terms of operational characteristics, unless operation in restricted regions permits the use of average operational characteristics.

The design of an AGC loop can be done completely in the frequency domain, starting with the desired closed loop frequency response and going back to the sequence of NICHOLS-BODE plots, appropriately selecting the poles and zeros of the loop. If some poles and zeros already exist in the physical system in the form of time constants in the transmission of signals around the loop (as is usually the case) then they must be taken into account in such a manner that considered together with the chosen filters, the loop gives the desired closed loop frequency response.

Now from the point of view of the frequency response of an AGC system, after a little consideration it becomes clear, that the loop acts as a non-linear filter.

If the closed loop frequency response of the third order sy tem considered in Appendix B was drawn, it would be easy for one to see that it is nothing but a band-pass filter.

Another way of looking at the AGC action in terms of the closed loop frequency response is the following:

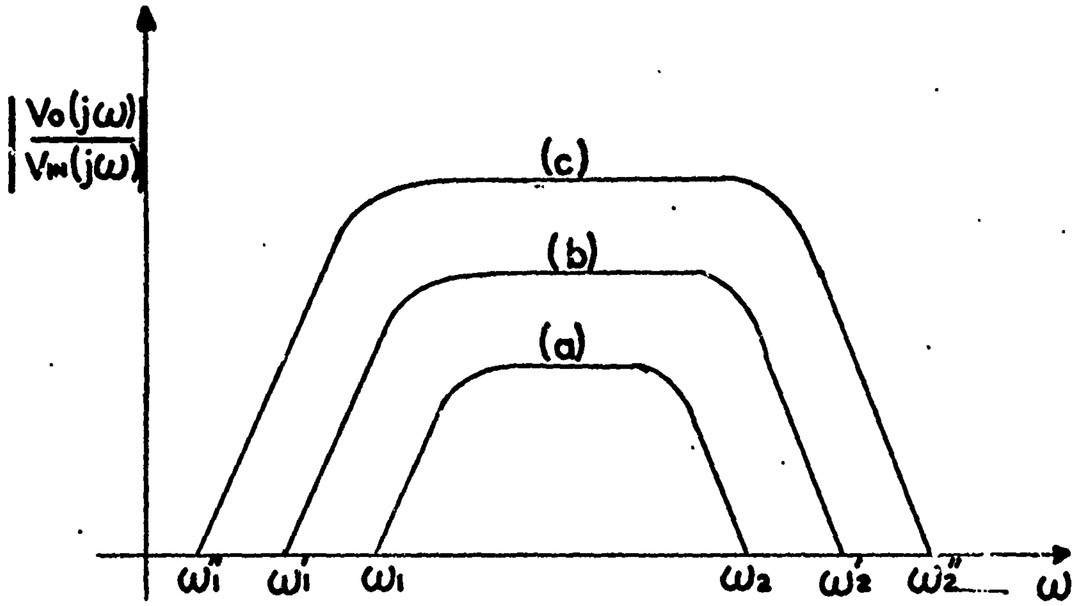


Figure 8. Closed-loop Frequency Response of Type-O-Systems.

The input frequency which has to be regulated must lie outside the passband of the system. Any frequency located inside this frequency band simply passes undisturbed by the regulating action. So if some frequencies must pass the AGC without suffering any reduction in their amplitudes, like the modulation frequencies in receivers, they must be located inside the frequency band of the loop.

The sketches in Fig. 9 show the sequence BODE (open)-NICHOLS-BODE (closed) of a receiver's AVC system. Sketch (c) indicates the closed loop frequency response, which is a H.P. filter and the frequencies that must be suppressed must lie outside of its band.

The justification of the sketches comes from the following crude derivation. Suppose the feedback filter is $H = \frac{p}{s+p}$ then

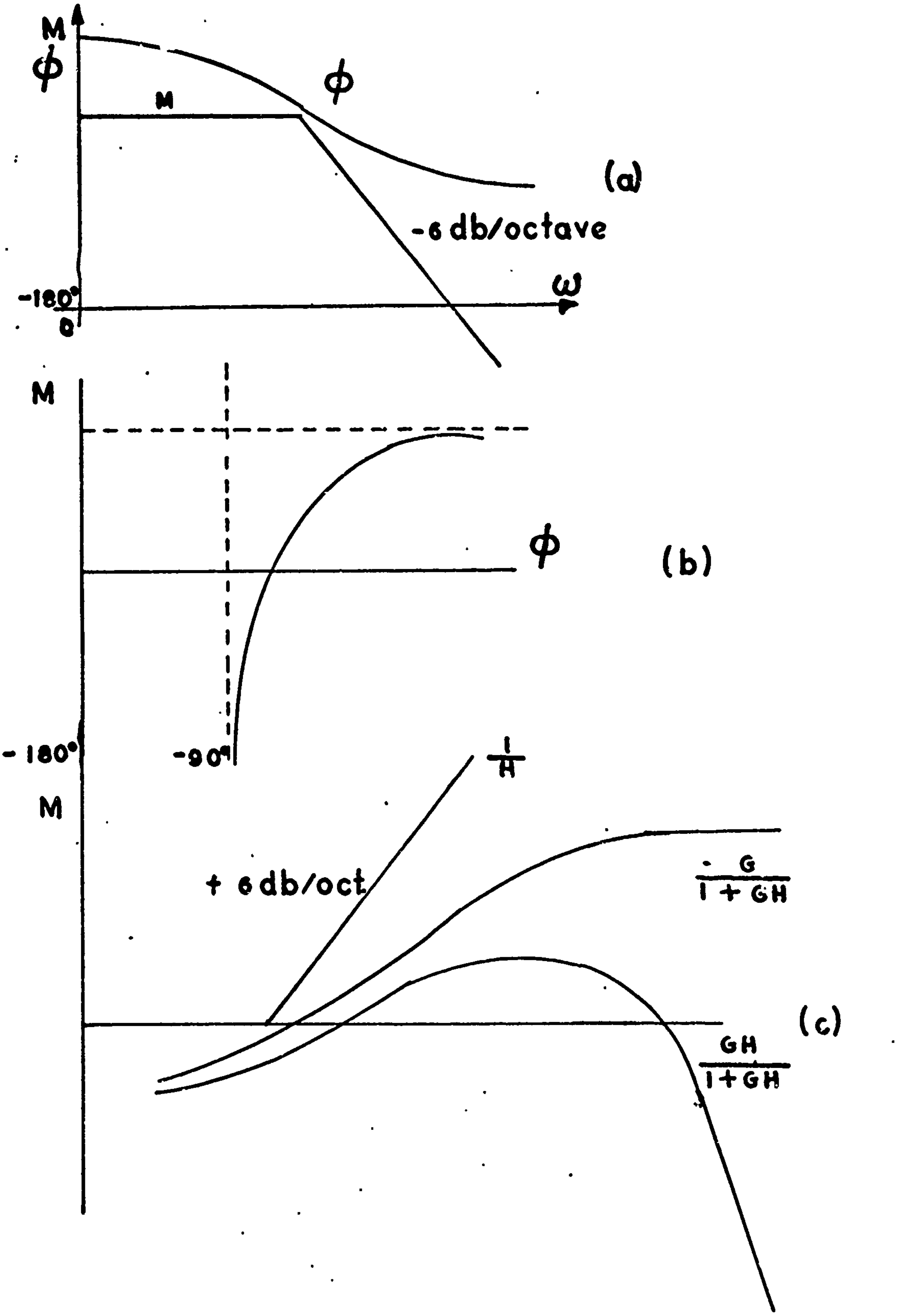


Figure 9. BODE (open)-NICHOLS-BODE (closed) Sketches of an AGC Loop with One Pole in the Feedback Path.

$$\frac{G}{1+GH} = \frac{k}{1+\frac{kp}{s+p}} = \frac{k(s+p)}{s+p+kp} = \frac{kp}{p+kp} \cdot \frac{\frac{1}{p}\, s+1}{\frac{1}{p+kp}\, s+1}$$

which is the transfer function of a .ead" filter since obviously $p < p+kp$.

It may be the case however that regulation action is desired throughout a wide range of frequencies and it is not desired on a single frequency or a very norrow frequency band located within the wide spectrum.

Such a situation may arise in an AGC system designed for a RADAR application.

In order for this to be accomplished two things have to be considered.

1. The loop gain over the desired frequency band must be kept less than 0 db in order for the regulation action not to take place.

2. The phase shift must remain constant in order for useful information not be be lost.

This situation may be faced as follows:

If the dynamics of the system (poles and zeros) are all included in the feedback path then the closed loop phase is given by

$$\angle \frac{KH(jw)}{1+KH(jw)} \cdot \frac{1}{H(jw)} = \angle \frac{K}{1+KH(j\omega)} = -\angle 1+KH(j\omega) \qquad (3\text{-}71)$$

where K is the loop gain.

Figure 10 shows the Nyquist sketch of such a system for two different values of the loop gain.

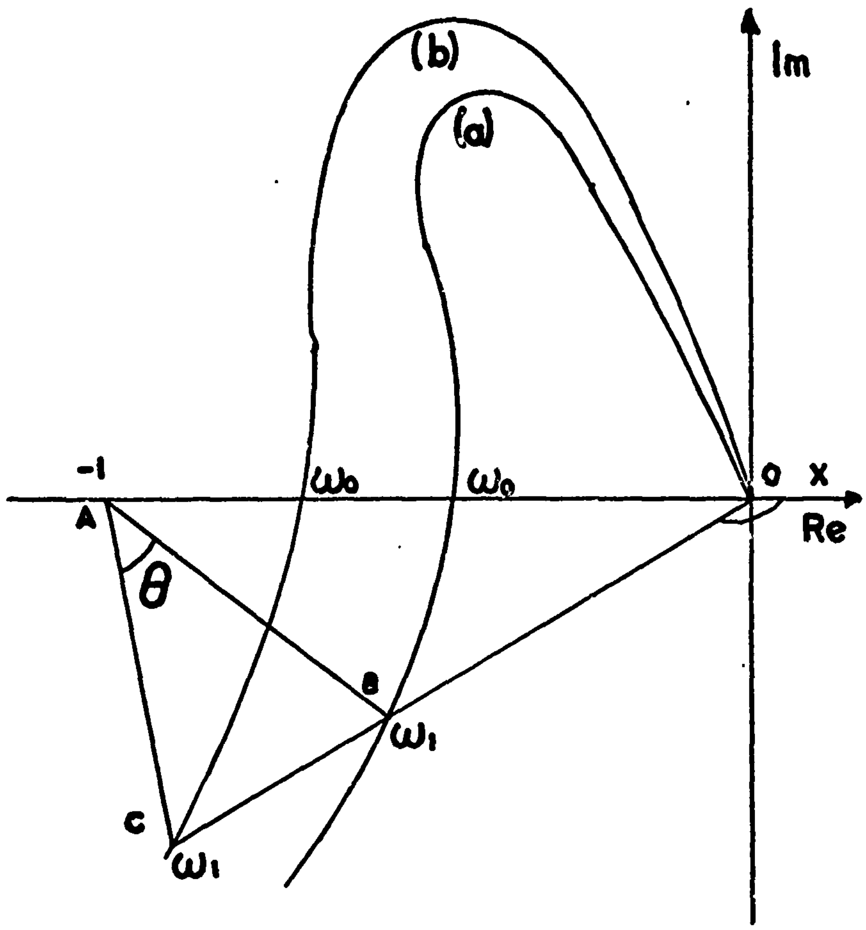


Figure 10. Phase Shift of 1+H(jw).

It is seen there that increasing the loop gain in system (a) results in system (b) without affecting the open loop phase shift (angles XOB and XOC for the two systems correspondingly). Since the system was assumed to have dynamics only in the feedback path the closed loop phase shift was derived as $- \underline{/1+H(j\omega)}$ and it is given by the angles OAB and OAC for the original and with increased gain systems correspondingly. Therefore it is observed that changingthe loop gain the closed loop phase shift changes by an amount θ. There exists however one frequency which

is not affected by the loop gain change as far as the closed loop phase shift is concerned and for which the loop gain remains less than 1. This is the phase crossover frequency and changing the loop gain simply slides it on the real axis producing a zero phase shift of the closed loop response. So with this technique incorporating into the feedback path of the AGC loop lag and/or lead filters to provide an open loop phase shift of -180° at and near the intelligence frequency gives an approximate zero closed loop phase shift for the desired band and an exact zero for one specific frequency.

One approach for accomplishing this goal is to attenuate $|KH(j\omega)|$ with a null over the required frequency band so that the maximum closed loop phase will be limited to a small value regardless of the phase of $KH(j\omega)$.

Figure 11 shows again the Nyquist plot of a system in which the quantity $|KH(j\omega)|$ has been made very small for a narrow band of frequencies.

It is seen that if the magnitude curve for this band of frequencies stays smaller than a predetermined constant value $|KH(j\omega_c)|$, then the maximum closed loop phase in the band ω_x-ω_y is ϕ_{max} and if $|KH(j\omega_c)|$ is very small ϕ_{max} remains close to zero. Now since $|KH(j\omega_c)| \ll 1$

$$\tan \phi_{max} = \frac{|KH(j\omega_c)|}{1+|KH(j\omega_c)|} \approx |KH(j\omega_c)| \tag{3-72}$$

From this, the required attenuation can be calculated. If for example it is required that ϕ_{max} be maintained less

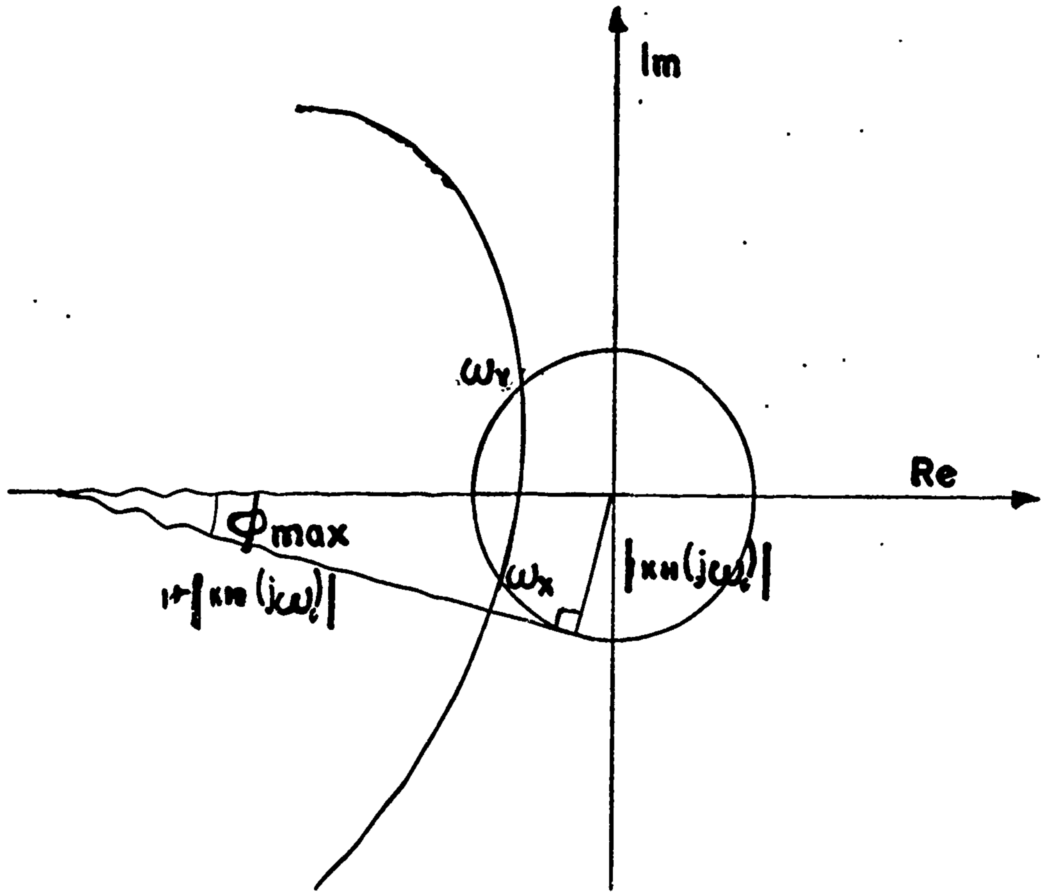


Figure 11. Closed Loop Phase Shift for $|KH(j\omega)|$ Made Very Small.

than 2° then the $|KH(j\omega_c)|$ at the intelligence frequency must be

$$|KH(j\omega_c)| < \tan 2° = .03492 = -29.14 \text{ db} \tag{3-73}$$

The null over a specified frequency can be obtained by a notch filter an example of which is a parallel - T network shown in Figure 12.

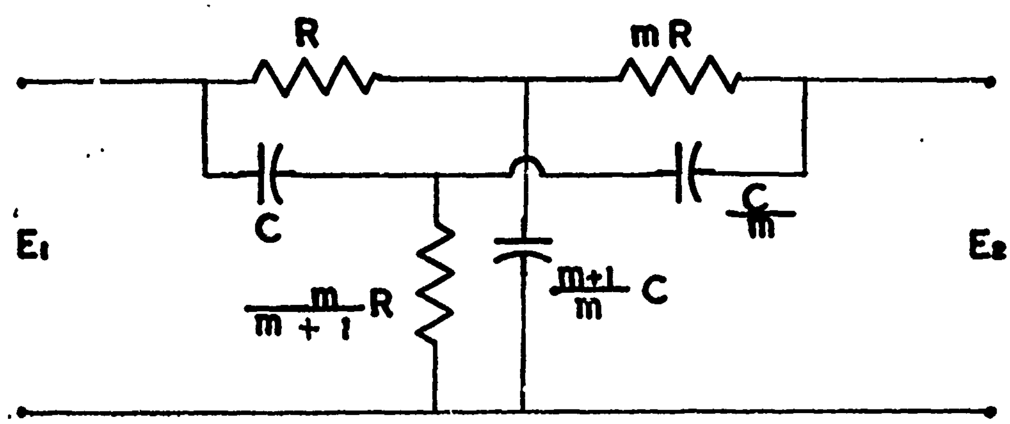


Figure 12. Parallel-T Null Network with Symmetry Pattern m.

The voltage transfer function for this network is

$$\frac{E_2}{E_1} = \frac{\left(j\frac{\omega}{\omega_c}\right)^2 + 1}{\left(j\frac{\omega}{\omega_c}\right)^2 + \left(j\frac{\omega}{\omega_c}\right)\left(2 + \frac{2}{m}\right) + 1} \tag{3-74}$$

where

$$\omega_c = RC$$

E. INPUT-OUTPUT CHARACTERISTICS

The static performance of an AGC system is often shown as the regulation curve of the steady-state output signal amplitude against input signal amplitude.

The following is an approach to derive a useful relation between the loop gain and the curves. In the derived equation (3-57)

$$\text{Loop gain} = G_1 G_2 G_3 \overline{V_{in}} \frac{d[N(v)]}{dv} \tag{3-57}$$

the factor $G_1\overline{V_{in}}$ can be replaced

$$\text{Loop gain} = G_2 G_3 \frac{\bar{V}o}{N(v)} \frac{d[N(v)]}{dv} \qquad (3\text{-}75)$$

if $V_{REF} = 0$, $G_2 G_3 \bar{V}o = -\bar{v}$ and substituting in the above equation

$$\text{Loop gain} = -\frac{\bar{v}}{N(v)} \frac{d[N(v)]}{dv} \qquad (3\text{-}76)$$

where the minus sign cancels the negative value of v which is assumed for this development.

Equation (3-76) says that in an "undelayed" system the loop gain depends entirely on the non-linear gain characteristic and the operating control voltage and is independent of the amplification around the loop.

The regulation curves are now drawn applying the following procedure.

For each output amplitude, $\bar{V}o$, the produced control voltage "$\bar{v}$" may be computed. From the non-linear characteristic, the corresponding gain $N(\bar{v})$ is found. Then the input signal is given as the ratio of the assumed output divided by the product $G_1 \cdot N(v)$ and a point is plotted defined by the pair of values Vo, Vin.

Figure 13 shows a typical non-linear gain characteristic. Since it is difficult, almost meaningless to use linear scales, logarithmic scales have been used and input and output are plotted in db.

Assuming $G_1 = 1.0$, $G_2 = 1.0$ and $G_3 = 1.0$, the solid regulation curves of Figure 14 were drawn, for Vomin = 0 ($V_{REF} = 0$) and Vomin = 10. The effect of adding amplification in the feedback path is shown by the dotted curves for

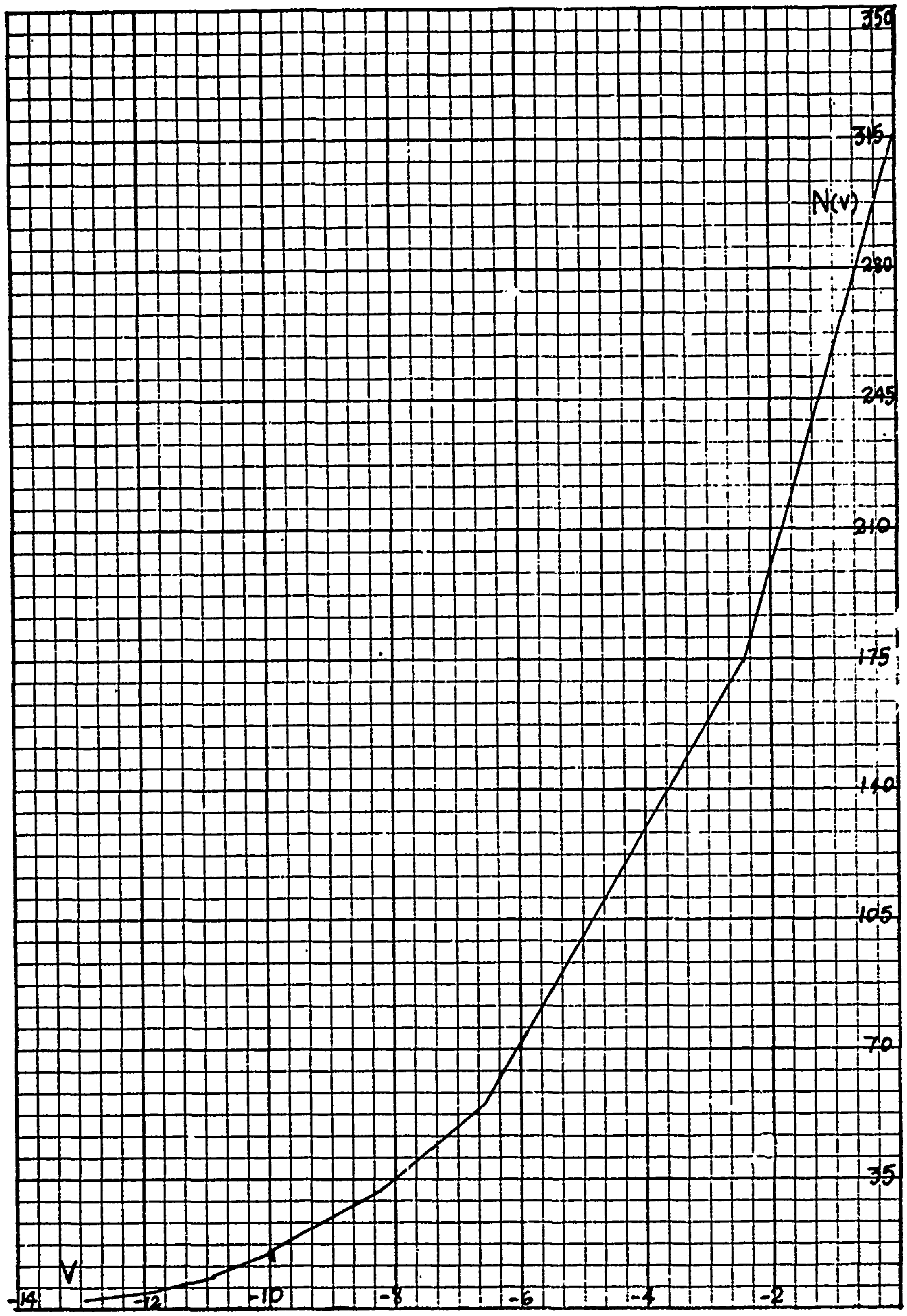


Figure 13. Typical Non-linear Gain Characteristic.

$G_2 = 10.0$ It is seen that in the zero-threshold case, $V_{REF}=0$ the entire output is simply reduced by a factor of 10 and the regulation is unimproved. For $V_{REF}=10$, only the amount by which the output amplitude exceeds the threshold is reduced by a factor of 10 and the regulation is greatly improved.

Following immediately are 4 tables with the calculated values used in the example. It is always assumed that the control voltage v cannot be positive and if G_2Vo is less than V_{REF}, v = 0 and N(v) = 315.

There is a unique relation between the regulation curves drawn in the Fig. 14 and the d.c. loop gain.

In equation (2-5) the factor μ is proportional to the input amplitude Vin, therefore if the input amplitude is substituted instead of μ the following formula is derived.

$$\frac{\frac{dVo}{Vo}}{\frac{dVin}{Vin}} = \frac{1}{1+\mu\beta} \tag{3-77}$$

which is entirely in accord with feedback theory. Equation (3-77) moreover can be written

$$\frac{d(\log_{10} Vo)}{d(\log_{10} Vin)} = \frac{1}{1+\mu\beta} \tag{3-78}$$

which means that if the regulation curves are drawn with logarithmic scales as in Fig. 14, then the slope of the curve at any point is

$$S = \frac{d(\log_{10} Vo)}{d(\log_{10} Vin)} \tag{3-79}$$

From equation (3-79) it follows that:

$$\text{Loop gain} = \quad = \frac{1}{s} - 1 \tag{3-80}$$

If the slope is unity then the loop gain is zero as in

TABLE I

Vomin = 10, G_2 = 1, V_{REF} = 10

Assumed Vo	Produced v	N(v)	Calculated Vin	Vo change in db
10	0	315	$\frac{10}{315}$ = .0318	0
12	-2	200	$\frac{12}{200}$ = .06	1.6
14	-4	130	$\frac{14}{130}$ = .108	2.94
16	-6	73	$\frac{16}{73}$ = .209	4.1
18	-8	35	$\frac{18}{35}$ = .515	5.12
20	-10	15.75	$\frac{20}{15.75}$ = 1.27	6
22	-12	4.5	$\frac{22}{4.5}$ = 4.9	6.86

TABLE II

Vomin = 0, G_2 = 1, V_{REF} = 0

Assumed Vo	Produced v	N(v)	Calculated Vin
2	-2	200	$\frac{2}{200}$ = .01
4	-4	130	$\frac{4}{130}$ = .0308
6	-6	73	$\frac{6}{73}$ = .0822
8	-8	35	$\frac{8}{35}$ = .228
10	-10	15.75	$\frac{10}{15.75}$ = .635
12	-12	4.5	$\frac{12}{4.5}$ = 2.67

TABLE III

Vomin = 10, G_2 = 10, V_{REF} = 100

Assumed Vo	Produced v	N(v)	Calculated Vin
10	0	315	$\frac{10}{315} = .0318$
10.2	-2	200	$\frac{10.2}{200} = .051$
10.4	-4	130	$\frac{10.4}{130} = .08$
10.6	-6	73	$\frac{10.6}{73} = .145$
10.8	-8	35	$\frac{10.8}{35} = .308$
11	-10	15.75	$\frac{11}{15.75} = .7$
11.2	-12	4.5	$\frac{11.2}{4.5} = 2.5$

TABLE IV

Vomin = 0, G_2 = 10, V_{REF} = 0

Assumed Vo	Produced v	N(v)	Calculated Vin
.2	-2	200	.001
.4	-4	130	.00308
.6	-6	73	.00822
.8	-8	35	.0228
1.0	-10	15.75	.0635
1.2	-12	4.5	.267

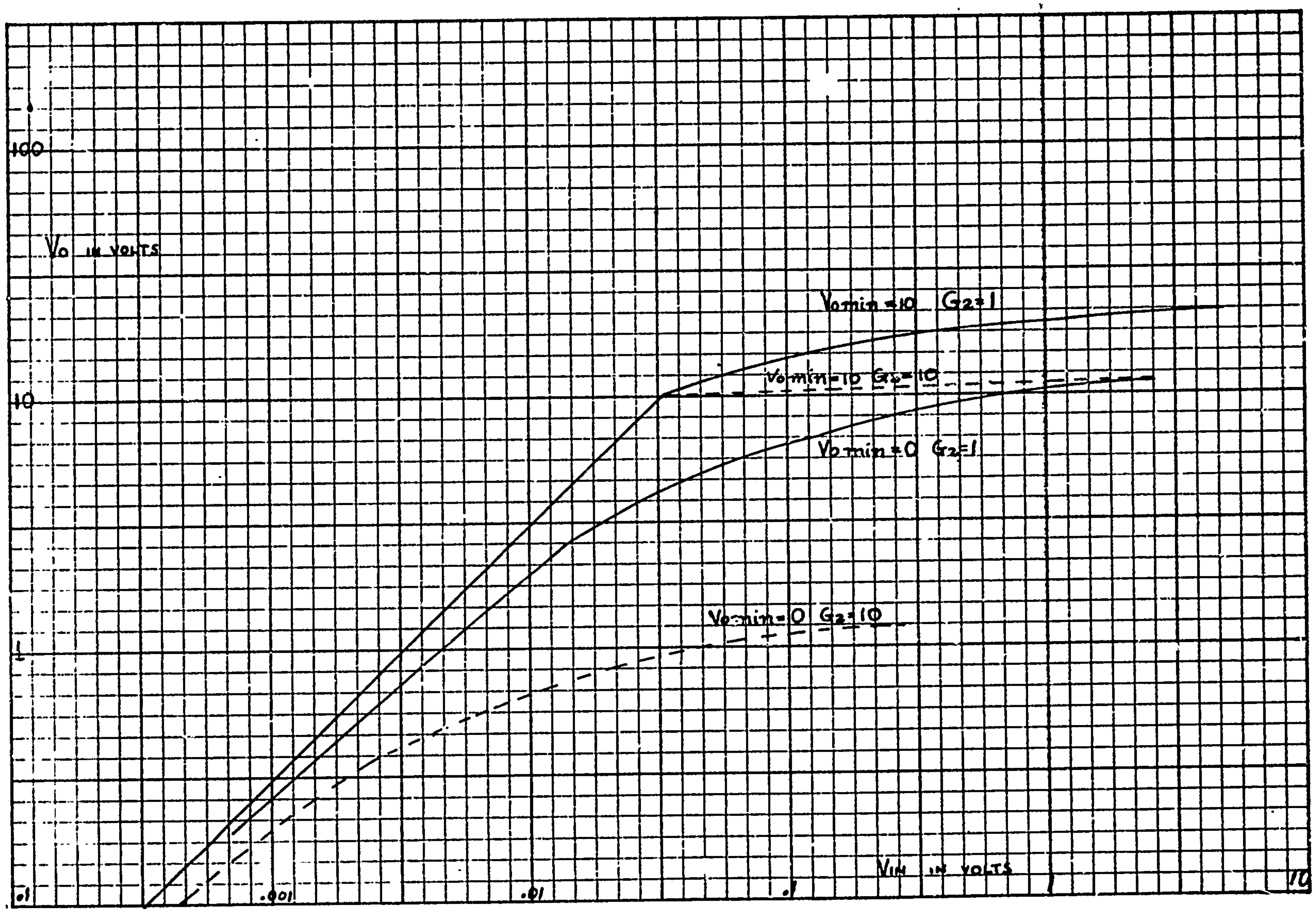


Figure 14. AGC Regulation Characteristics.

the case where $V_{REF} = 10$ for output amplitudes below 10.

When $V_{REF} = 0$, the slope of the regulation curve was nowhere affected by increasing G_2 from 1 to 10. Thus, the loop gain was unchanged as predicted by (3-76).

A slightly different approach to drawing the regulation curves can be applied utilizing the relations between input and output in db.

First the gain curve is expressed and plotted in db vs control voltage "v". Then the output changes in db are plotted on the same graph vs the produced control voltage. Then the input curve is derived using the relation

$$Vin(db) = Vo(db) - N(db) - G_1(db) \qquad (3\text{-}81)$$

For the example considered above the gain curve in db is shown in Fig. 15 as curve (a).

The output changes in db for the first case are plotted as curve (b) from column 5 of Table I.

The input is then derived shown as curve (c) with scale indicated at the left of the figure. Taking corresponding pairs of values it is easily shown that the upper part of the first solid curve of Fig. 14 is traced. The same procedure applied to the other cases yields the same transfer characteristics of Fig. 14.

F. DISTORTION

The AGC loop must maintain the nominal output amplitude constant and it does this by feeding back a signal proportional to the output amplitude and thus changing the gain by which the input signal is multiplied. If the amplifier characteristic is considered in terms of the output current

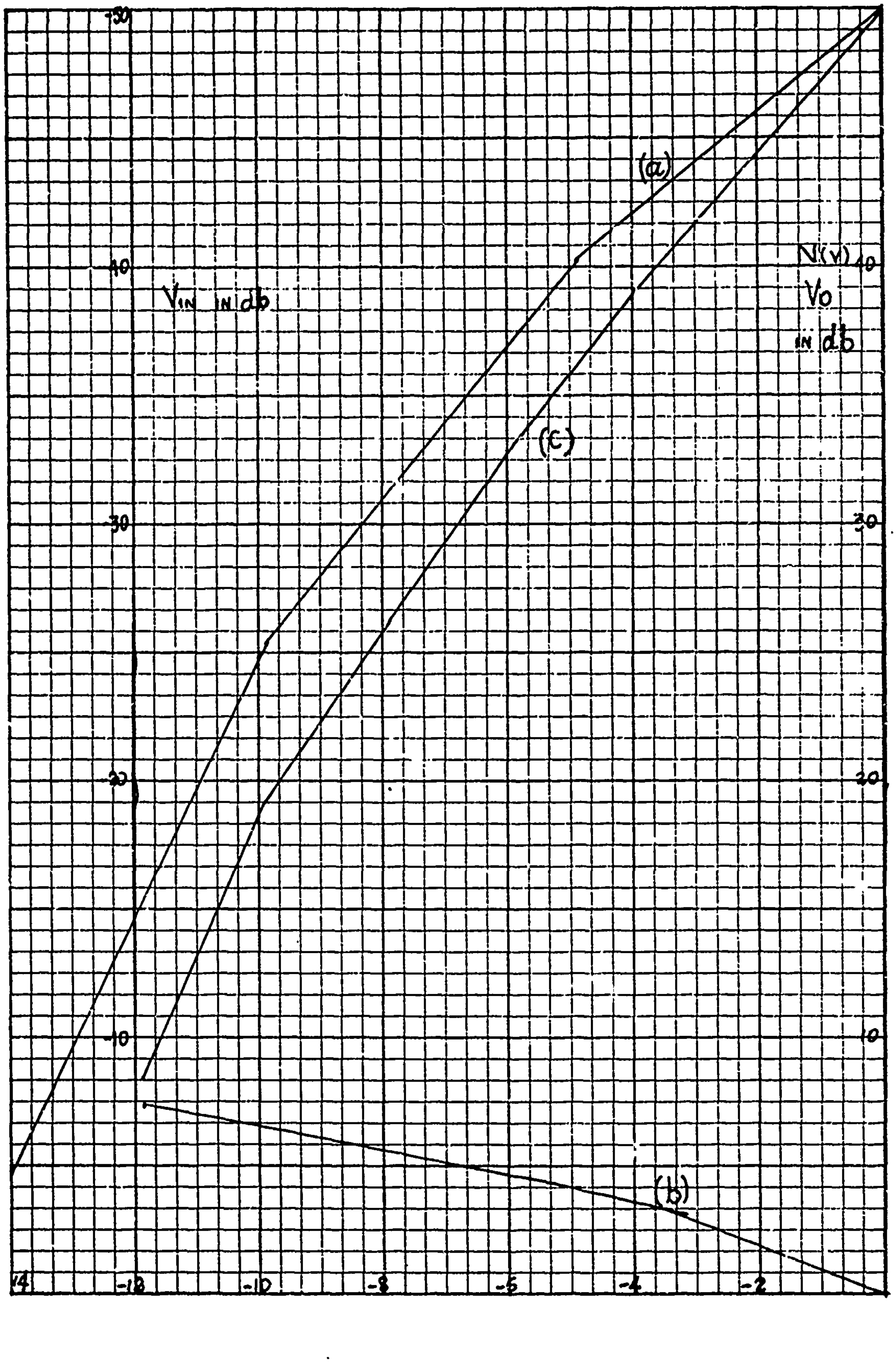


Figure 15. Relations Between Vin, Vo, and $N_{(v)}$ in db.

(i_p) vs the grid voltage (v_g) then the gain of the amplifier is the magnitude of the slope at the operating point. Fig. 16 shows how an incoming signal is amplified.

The gain is $N(v) = \frac{\partial i_p}{\partial v_g}$ which is the transconductance gm of the amplifier.

The AGC regulation is achieved by changing the bias voltage v_1 thus moving the operating point A on the transfer characteristic and obtaining steeper or shallower slopes depending on the variation of v.

Generally the non-linear transfer characteristic without AGC action, introduces distortion in the output signal,

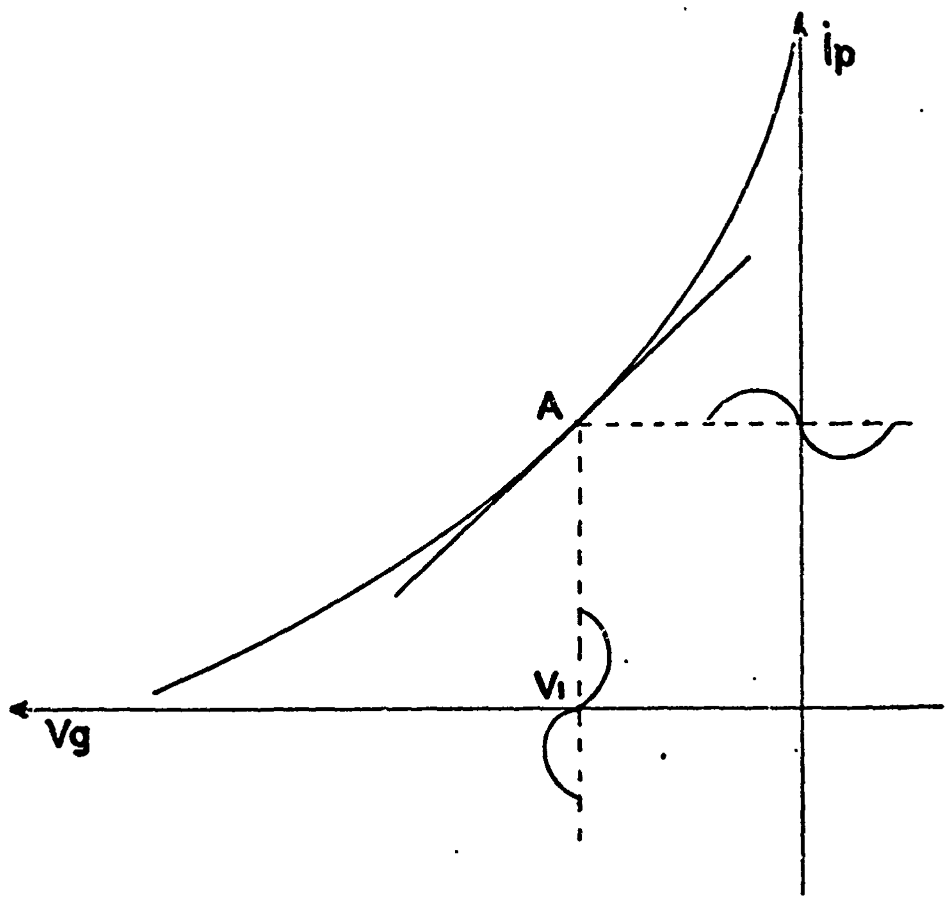


Figure 16. Typical Amplifier Characteristic.

which is easily calculated, assuming a power series representation of the transfer characteristic and an input signal, usually cosinusoid. The derivation of the formulas giving the second-third etc. harmonic distortion of the output signal can be found in almost all basic electronics texts.

The AGC action introduces an additive distortion which is rather difficult to predict.

Consider Fig. 17 which shows one sinusoidal carrier modulated by a sinusoid, passing through a delayed AGC loop with delay voltage V_{REF}. In 17(a) the modulation envelope is lower than V_{REF} so no AGC action takes place and the wave passes undistorted. In 17(b) the carrier peak voltage is equal to V_{REF} so whatever portion of the modulation envelope is above V_{REF} undergoes AGC action. In 17(c) the entire modulation envelope is above V_{REF} and so undergoes AGC regulation.

Hence the distortion is a function of the amplitude of the carrier as well as the modulation index m, and the AGC voltage v applied to the grid.

In order to determine the actual distortion through an AGC loop an accurate description of the amplifier transfer characteristic is required. If it is assumed that the transfer characteristic may be expressed as a power series in v, the grid voltage, then a good measure of the AGC distortion is the change of the modulation index m, of a sine wave modulated by a cosine wave.

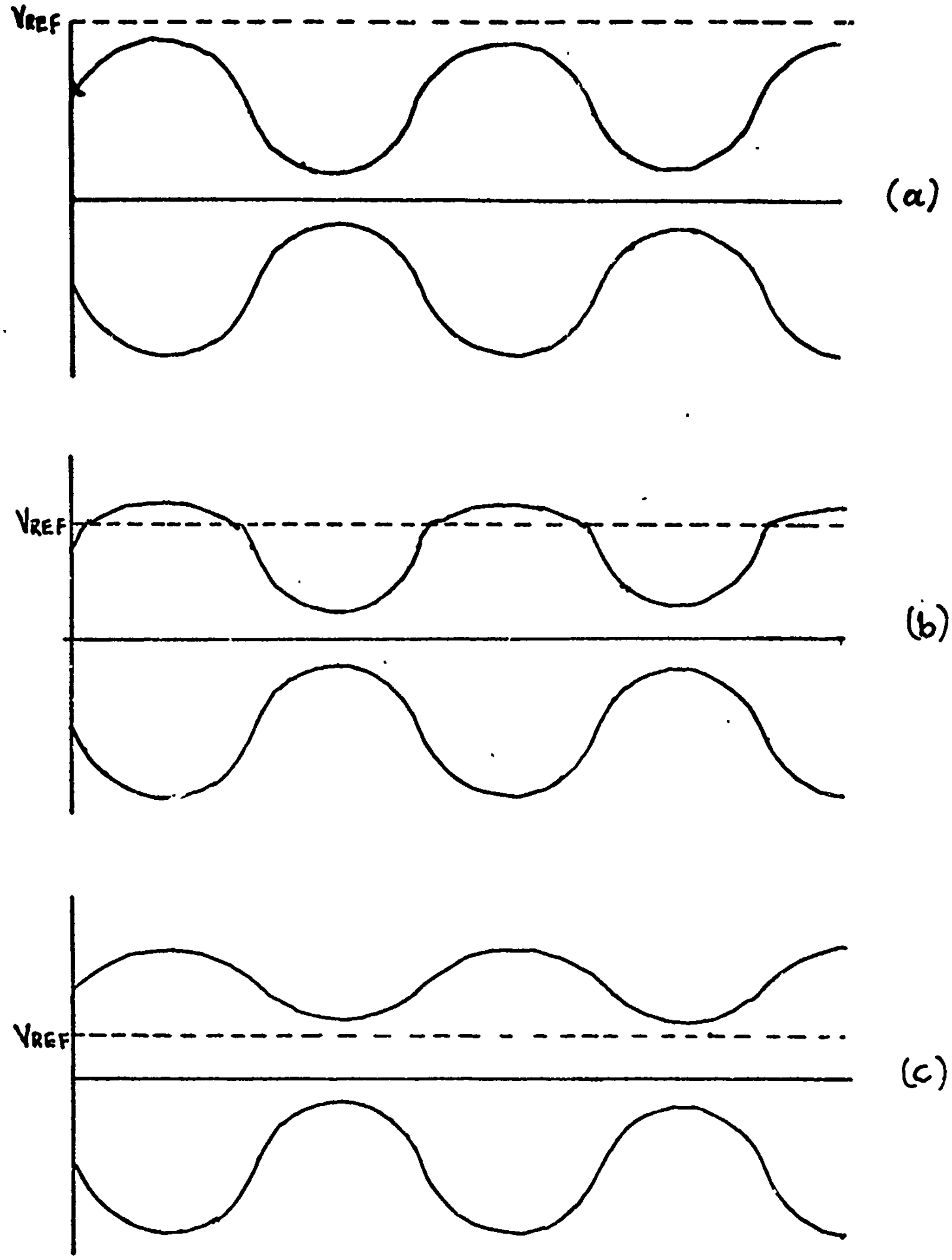


Figure 17. Various Stages of Input Signal Damping
Due to Biased AGC Loop.

(a) Modulation envelope less than bias voltage

(b) Carrier peak voltage equal to bias voltage

(c) Modulation envelope greater than bias voltage

Thus, assuming $Vin = A[1+m \cos \omega_m t] \sin \omega_c t$ the amplifier transfer characteristic given by $T(v) = a_o + a_1 v + a_2 v^2 + a_3 v^3$, the bias voltage $v = v_1$ and that the output is given in the form $Vo = B[1+m' \cos \omega_m t] \sin \omega_c t$ (all the other components generated by the non-linear action are assumed to be filtered out) then the relation between the modulation index in the output m' and in the input m is given by:

$$\frac{m'}{m} = 1 + \frac{\frac{9}{4}a_3 A^2(1-\frac{3}{4}m^2)}{a_1+2a_2v_1+3a_3v_1^2+\frac{9}{4}\,a_3A^2(1+\frac{3}{2}m^2)} \qquad (3\text{-}82)$$

Derivation of the formula is given in Appendix D. The derived formula is however a rather inconvenient measure of the distortion.

A less accurate one but easily evaluated by inspection of the gain curve vs v is the following

$$\frac{m'}{m} = \frac{1}{2}\left[1 + \frac{Nmax+Nmin}{2No}\right] \qquad (3\text{-}83)$$

where No is the gain corresponding to a bias voltage v_1 and Nmax, Nmin are the maximum and minimum gains corresponding to the trough and peak of the input waveshape respectively.

IV. DESIGN-CONSIDERATIONS

A. COMPENSATION-SHAPING OF TRANSIENT RESPONSE

It was shown in Section III-A that a characteristic equation can be derived for a linearized AGC loop, equation (3-54) which contains the non-linear factor of the loop gain. There were also expressed some special features of stability and performance of AGC loops regarding the non-linearity of the loop gain factor, and it was pointed out that conventional methods of design of linear loops are applicable in the case of AGC loops.

The approximate linear feedback loop corresponding to the derived transfer function Eqn. (3-34) is pictured in Fig. 18 where $No = Ao + A_1\bar{v}$ the nominal value of the amplifier's gain and $K = A_1\overline{Vin}\; G_1G_2G_3$ is the loop gain.

It is clear from the theoretical derivation of chapter III and from Fig. 18 that the stability of the AGC loop and the transient response for small signal operation depends entirely on the characteristic equation

$$1 + A_1\overline{Vin}\; G_1G_2G_3\; G_1(s)G_2(s)G_3(s) = 0 \qquad (4\text{-}1)$$

For the sake of illustration of stabilizing an AGC loop using linear feedback techniques the example of Appendix B has been worked in Appendix E. It has been proved that indeed the loop which was unstable for an input amplitude exceeding $\overline{Vin}$ = .11, now inserting a "lag" filter in the forward path it becomes stable for a range of input amplitude

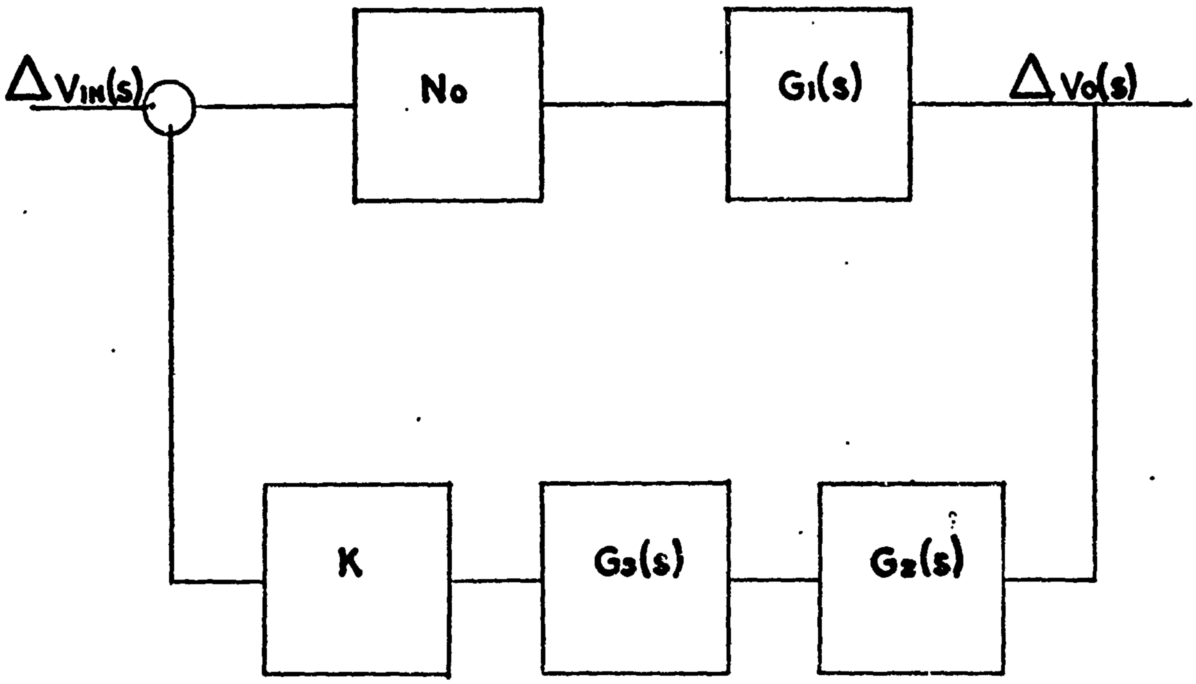


Figure 18. Block Diagram of Linearized AGC Loop.

exceeding the previous limit. In addition, interpreting the BODE plot as in conventional linear feedback theory the new limit of input amplitude is predicted above which the system again reaches a limit cycle.

Due to reasons explained in Chapter III-C, before the application of any input the gain of the amplifier is at its maximum value, thus producing the very large overshoot shown in the simulations. For two values of input amplitude, additional simulations were tried, keeping a bias voltage which could have been there from previous input, if a two time constant filter were in the feedback path, and thus having a reasonable initial value for gain.

The results compared with the ones obtained without bias show clearly how the maximum overshoot and the settling time have been reduced.

In Section III-A it was pointed out that since the loop gain of an AGC loop depends among others on the input amplitude, it is not easy to determine an operating point and from the location of the roots to determine the performance characteristics of the system. It is however possible to determine a region in which the roots of the system lie and so average performance characteristics can be derived for the particular range of input amplitude which places the roots in the above mentioned region.

Of course if the input amplitude remains constant the location of the poles is fixed and performance characteristics can accurately be predicted, in this case however the system does not function as an AGC loop because if the input amplitude remains constant there is no need for AGC action.

Another example has also been worked of an AGC loop having 5 poles (3 real, 2 complex) and 2 complex zeros. The root locus of this system (obtained by the computer) shows a pair of dominant complex poles near the imaginary axis thus creating undesirable effects on the transient response. Compensation has been made inserting a "lead" filter in the forward path and the results of the compensated loop are shown by the simulation to be very good.

B. FEATURES OF THE NON-LINEAR GAIN CHARACTERISTIC

The whole philosophy behind the AGC action is that there exists an amplifier the gain of which is variable. The input amplitude changes because of various reasons and is multiplied by the instantaneous gain of the amplifier.

There is a sensing device which feeds back a measure proportional to the output amplitude and compares it with a preset reference level. The difference between those two acting as an error signal controls the gain of the amplifier in such a manner that the output level is kept as constant as possible.

In this section answers will be attempted to two questions concerning the Gain characteristic of the amplifier.

First if a voltage controlled amplifier is given the gain characteristic of which is known and it is going to be matched to a specific plant to produce a constant amplitude output despite the variations in the input amplitude, what is the maximum achievable suppression of input fluctuations and how should the various linear gains and components be arranged around the loop? Second, if a specific degree of regulation and performance characteristics are desired, what will be the special features in the gain characteristic of the voltage controlled amplifier in order for the specifications to be met?

The main results of the analysis of Section III helpful in this development here are the following:

i. The regulation achievable by an AGC system is proportional to the loop gain, and if a specific regulation is asked for, the required loop gain can be calculated, and vice versa, using equation (3-64) which is repeated here:

$$\text{Loop gain} = \frac{0.115\ G_2\bar{V}o\ [\text{Gain change in db}]}{Vomax - Vomin} \qquad (4\text{-}2)$$

ii. The loop gain is determined by the physical parameters of the system and is given by (Loop gain) = (Linear gain) x (Input amplitude) x (Slope of amplifier gain characteristic) (4-3)

iii. The linear gain in the feedback path G_2G_3 is given by

$$G_2G_3 = \frac{vmax - vmin}{Vomax - Vomin} \qquad (4\text{-}4)$$

iv. The reference voltage is specified precisely by the requirements and the physical parameters of the system and is given the formula

$$V_{REF} = G_2Vomin + vmin \qquad (4\text{-}5)$$

and is the means of forcing the loop to operate in the chosen region of the gain characteristic.

Suppose that the given gain characteristic is expressed as the logarithm of the gain vs bias voltage and is represented by the graph of Fig. 19. Suppose also that the input amplitude Vin is expressed in db and Vinmax - Vinmin = 20 db and $G_1 = 1$.

If the performance of the AGC loop is specified as the need of suppression of the input variations by 10 db then in essence a pair of values of the Gain Nmax, Nmin must be chosen such that Nmax - Nmin = 10 db. If all the remaining components and gains of the loop are arranged in such a manner that finally Vomax = Vinmax Nmin and Vomin = Vinmin· Nmax or expressed in db. Vomax = Vinmax + Nmin and Vomin = Vinmin + Nmax then obviously the output signal amplitude will have variations 10 db less than the corresponding variations of the input amplitude and the main goal of the AGC loop has been achieved.

It is worth noting at this point that nothing has been said about the actual values of Nmax and Nmin as long as they have a difference of 10 db, and of course nothing has been said about the actual levels of input and output amplitudes, they are just discussed comparatively mentioning only the relative differences. This means that the input amplitude can vary from 10 v to 100 v (20 db) or 100 v to 1000 v (20 db) and the corresponding output amplitudes can be for example 10 v to 31.6 v (10 db) or 100 v to 316 v (10 db) or any values having a difference of 10 db or in other words whatever values satisfy the relation $\frac{Vomax}{Vomin} = 3.16$.

Thus the actual values of Nmax, Nmin are not of special interest in the design of the AGC loop, since they only specify the actual value of the output voltage and this is not of primary importance. On the other hand suppose that the output of the AGC loop is well regulated as far as the

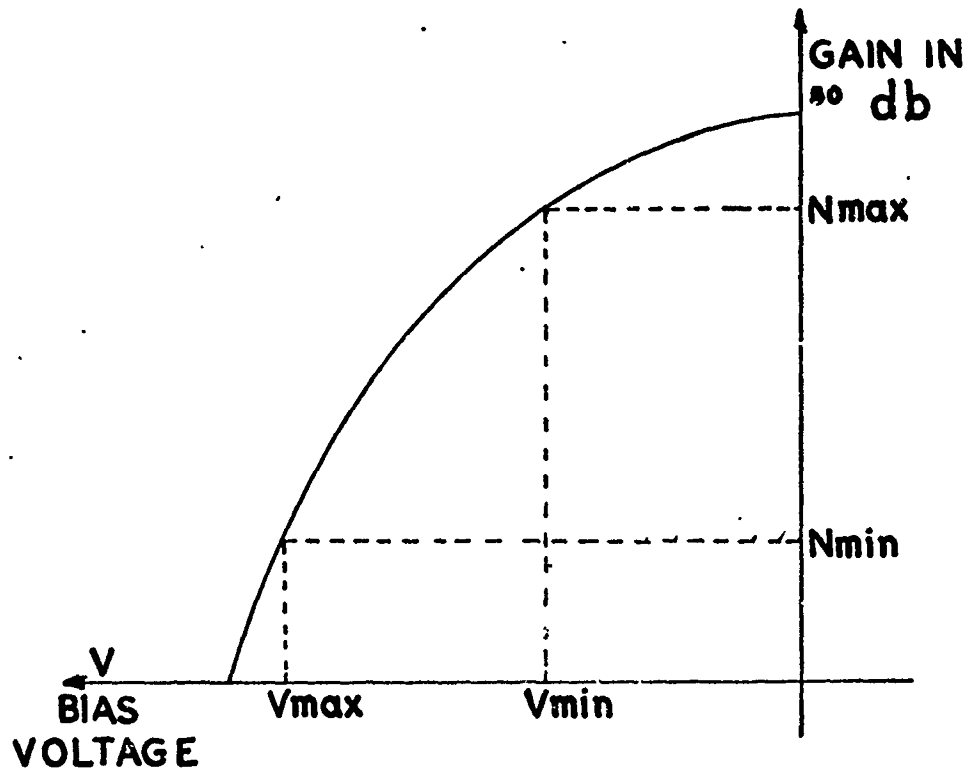


Figure 19. Gain Characteristic of Voltage
Controlled Amplifier.

relative differences is concerned but the level of the output voltage does not fit into the entire system because it is too high or too low to be used. In this case it can easily be brought to the desired level passing it after the AGC loop through a linear attenuator or amplifier or using an appropriate linear gain G_1.

The irrelevancy of the actual Nmax, Nmin provides the flexibility of choosing the most convenient pair of values enclosing the operating region on the gain characteristic.

Various features of the gain characteristic may suggest a choice of one operating region instead of another. For example if the slope of the characteristic is very high then the difference of the extreme values of the bias voltage

vmax - vmin is very small. This compared with the signal levels of the whole system may lead to inconvenience and more susceptibility to noise and random disturbances. To this must be added that the effects of the transient response have not been considered, the discussion here being restricted to the steady-state condition. As an example, if a system is designed to regulate an input voltage varying from 10 to 50 volts into an output voltage varying from 20 to 22.5 volts then obviously, if the slope of the chosen operating region is such that vmax - vmin is of the order of 1 or 2 volts of less, this would create a rather unreliable system susceptible to small fluctuations of 0.5 volts with a jittering output. Another important factor in choosing the operating region on the gain characteristic is the realization of the linear feedback gain given by equation (4-4). It is seen that if the difference of the extreme values of bias voltage is smaller than the output voltage range then an attenuator instead of an amplifier is needed and if the bias voltage range is too small the realization of the feedback linear gain may be impractical.

The forward linear gain G_1 is a means of designating the actual level of the output signal and bring it in the system's requirements if not so. On the other hand it can have its share of the linear gain requirement of the system. Suppose that the bias voltage range turns out to be 10 volts and the output range is 0.1 volts. Then applying equation (4-4) the feedback gain G_2G_3 must be 100 which perhaps is

too high, or the output level is considered too low. A forward gain G_1 can be added to the system raising the output level (but keeping the same proportional regulation $\frac{Vomax}{Vomin}$) and then G_2G_3 will be $\frac{100}{G_1}$. In this respect equation (4-4) can be transformed as

$$(\text{Linear gain}) = G_1G_2G_3 = \frac{vmax - vmin}{v_2max - v_2min} \qquad (4\text{-}6)$$

where v_2max and v_2min are the extreme values of the amplifier output (not the loop output).

Finally the reference voltage V_{REF} provides a means of realizing the chosen bias of the system. If all the rest of the components of the system have been decided considering the above mentioned factors, then the reference voltage is calculated substituting numbers in equation (4-5) and it is the tool which will conduct the output amplitude range Vomax - Vomin into the chosen bias range vmax - vmin.

A few words are pertinent here as far as equation (4-5) is concerned. The equation is correct for the configuration shown in Fig. 19, that is, for negative bias voltage and gain characteristic with positive slope or positive bias voltage and negative slope as in Fig. 20.

This is easily understood because the measure of the minimum output voltage has to correspond to the minimum bias voltage thus producing the maximum gain so V_{REF} - Vomin G_2= vmin and V_{REF} = Vomin G_2 + vmin which is equation (4-5) and where the vmin is added or subtracted depending on its sign.

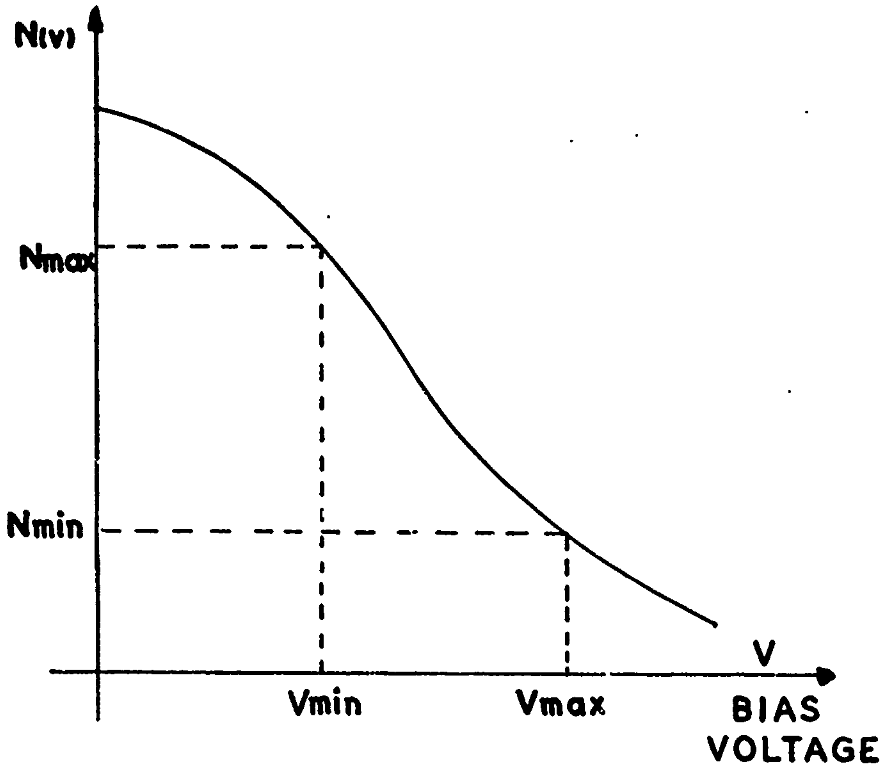


Figure 20. Gain Characteristic with Positive Bias Voltage and Negative Slope.

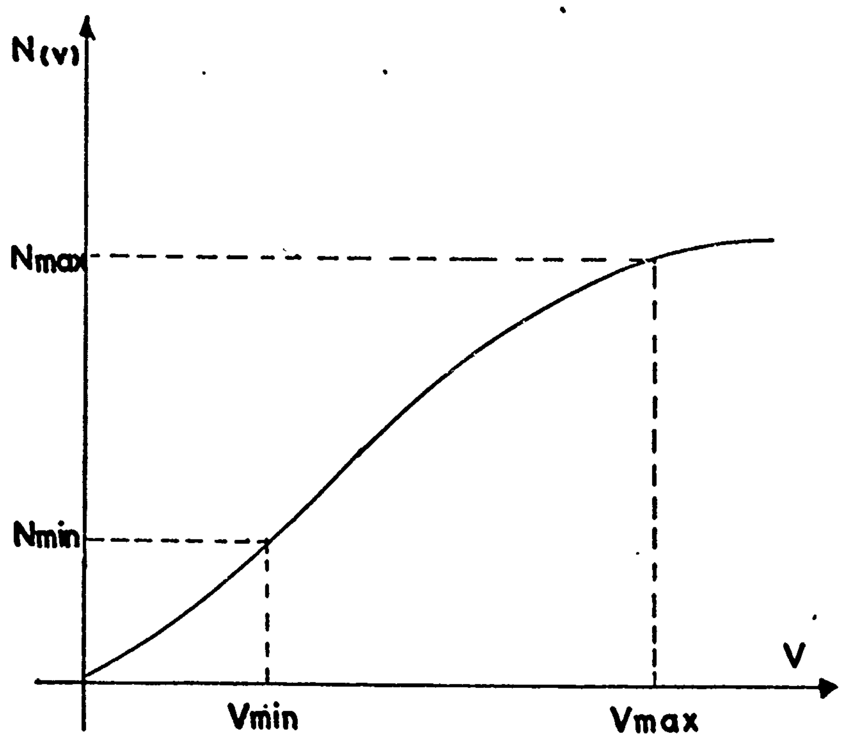


Figure 21. Gain Characteristic with Positive Bias Voltage and Positive Slope.

If however the signal controlled amplifying device has a gain characteristic like the one shown in Fig. 21 that is positive bias voltage and positive slope of the characteristic then applying the same idea as before

$$V_{REF} - Vomin\ G_2 = vmax$$

and

$$V_{REF} = Vomin\ G_2 + vmax \tag{4-7}$$

As a conclusion of the whole above development comes the fact that the regulation achievable by the AGC loop depends on the range of gains included in the operating region of the gain characteristic. So if the maximum possible regulation is desired all of the gain characteristic can be used. For example, for the characteristic of Fig. 19 a regulation of 50 db is possible although other considerations as distortion of the input signal may be prohibitive.

Now in order to achieve a specific regulation say 20 db less than the fluctuations of the input signal, a gain characteristic having at least this range is required. If an amplifier with greater range of gain is available then choice of operating region taking into account other factors can be made.

C. CONSTRUCTION OF THE NON-LINEAR CHARACTERISTIC-DISTORTIONLESS CHARACTERISTIC

As was previously observed no restriction was put on the actual shape of the graph of gain versus bias voltage, except that the range of the gain must be large enough

to provide the required amount of regulation in the output.

When the voltage controlled amplifier is given, the gain versus bias voltage curve can be easily gotten and the problem is restricted to choosing an appropriate working region on the curve, and to assigning the various parameters around the loop in order to restrict the operation to this region. Thus, the desired regulation in the output is obtained. If, however, one is required to choose the best gain versus bias voltage curve, or even to construct by a diode function generator the best curve suited for the AGC loop, various considerations must be taken into account.

In this section an attempt will be made to derive the shape of two general curves best suited for AGC action, covering two general cases of specifications.

As a first case consider the specification of varying the regulated output analogously to the input variations. The specification is better depicted in Fig. 22 where a linear relationship is shown between input and output when both are expressed in db.

Steady state calculations are used as shown in the analysis in Chapter III. The AGC loop block diagram is shown in Fig. 23. Then

$$\overline{V}o = \overline{Vin}\, G_1\, N(v) \qquad (4\text{-}8)$$

and transforming into db

$$20 \log\overline{V}o = 20 \log\overline{V}in + 20 \log G_1 + 20 \log N(v) \qquad (4\text{-}9)$$

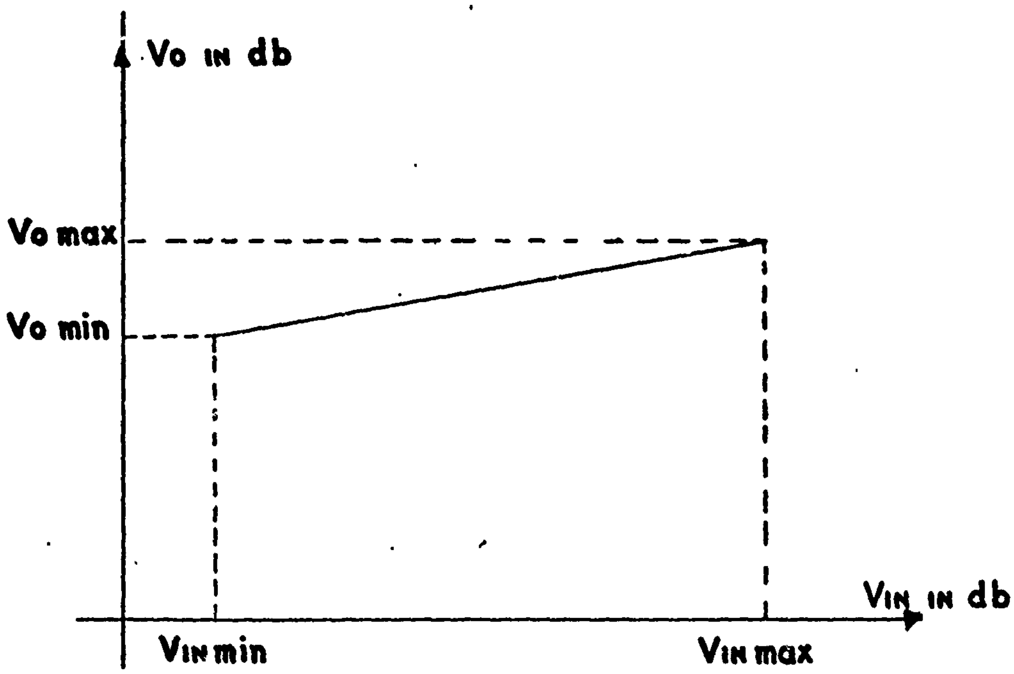


Figure 22. Linear Relationship Between Input and Output When Both are Expressed in db.

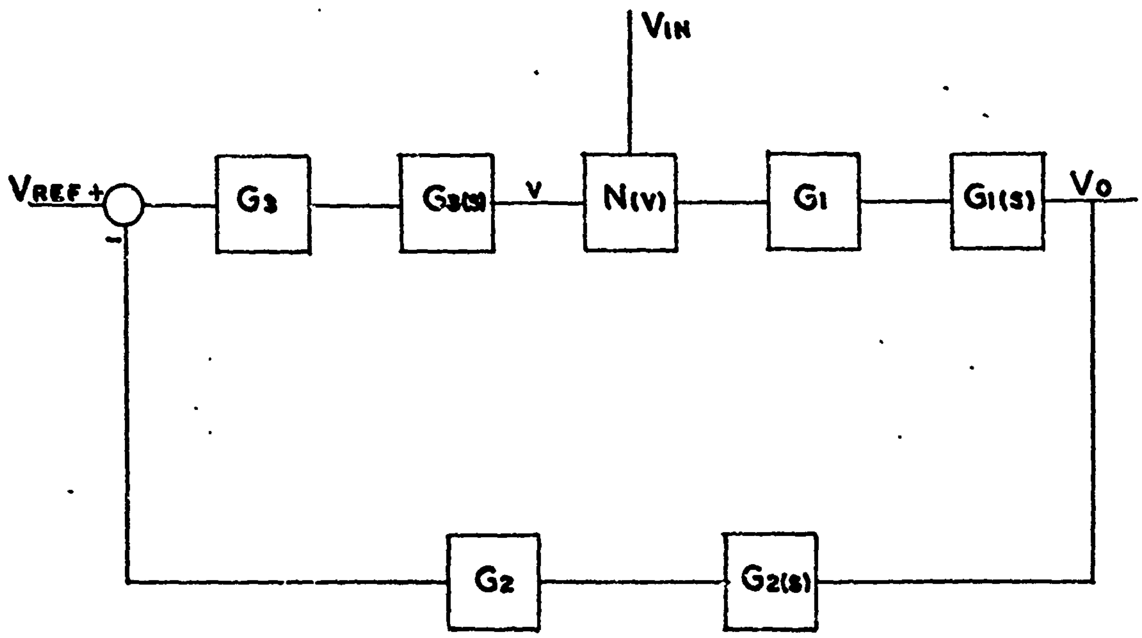


Figure 23. Block Diagram of AGC Loop.

assuming for convenience that $G_1 = 1.0$

$$\overline{V}o(db) = \overline{Vin}(db) + N(v)(db) \tag{4-10}$$

which is the straight line shown in Fig. 22. Equation (4-10) can be written as:

$$N(v)(db) = \overline{V}o(db) - \overline{Vin}(db) \tag{4-11}$$

Suppose now that the loop is set to regulate A db of input amplitude variations into 1 db of output amplitude variations and N points of the straight line of Fig. 22 are to be calculated.

Since the relationship between input and output has been taken as linear, at each increment $\frac{A}{N}$ of input amplitude, corresponds an output amplitude increment $\frac{1}{N}$. Substituting these two values into (4-11)

$$N_n(db) = -n(\frac{A}{N} - \frac{1}{N}) = -n\,\frac{A-1}{N} \tag{4-12}$$

where $n = 0,1,2,\ldots,N$

Thus successive values of gain can be calculated substituting integer values for n into (4-12). If the forward gain G_1 is different than 1.0 the equation (4-12) takes the form:

$$N_n(db) = -n\,\frac{A-1}{N} - G_1(db) \tag{4-13}$$

The calculated values of gain can be plotted against an arbitrary division of bias voltage in equal intervals and they produce a straight line with slope

$$s = \frac{N_n - N_{n-1}}{V} \tag{4-14}$$

where V is the arbitrary increment in bias voltage.

An example of such straight lines calculated for 5, 10, 20, 30 and 40 db variations in input amplitude, regulated into 1 db of output amplitude variations are shown in Fig. 24 plotted against a normalized bias voltage length of - 1.0.

Now these curves have the general equation:

$$20 \log N(v) = s \cdot v - G_1 (db) \qquad (4\text{-}15)$$

(In Fig. 24 G_1 has been taken as 1.0.)

where s is the slope of the straight lines. Taking the antilogarithm

$$N(v) = 10^{\frac{sv - G_1 (db)}{20}} \qquad (4\text{-}16)$$

Therefore in the case of the particular specification considered here, the plot of the non-linear gain versus the bias voltage is a logarithmic function the analytical expression of which is given in equation (4-16). Plots of equation (4-16) corresponding to the straight lines of Fig. 24 are given in Fig. 25 for a normalized gain of 1 and a normalized bias voltage length of -1.

The use of these curves is straightforward. If a specific regulation is needed the appropriate curve is selected or the one that gives immediately higher regulation. The scales are stretched according to the specific values of output amplitudes and parameters of the system. Curve (E) of Fig. 25 can cover all the lower cases with the disadvantage of having higher slope which makes it more

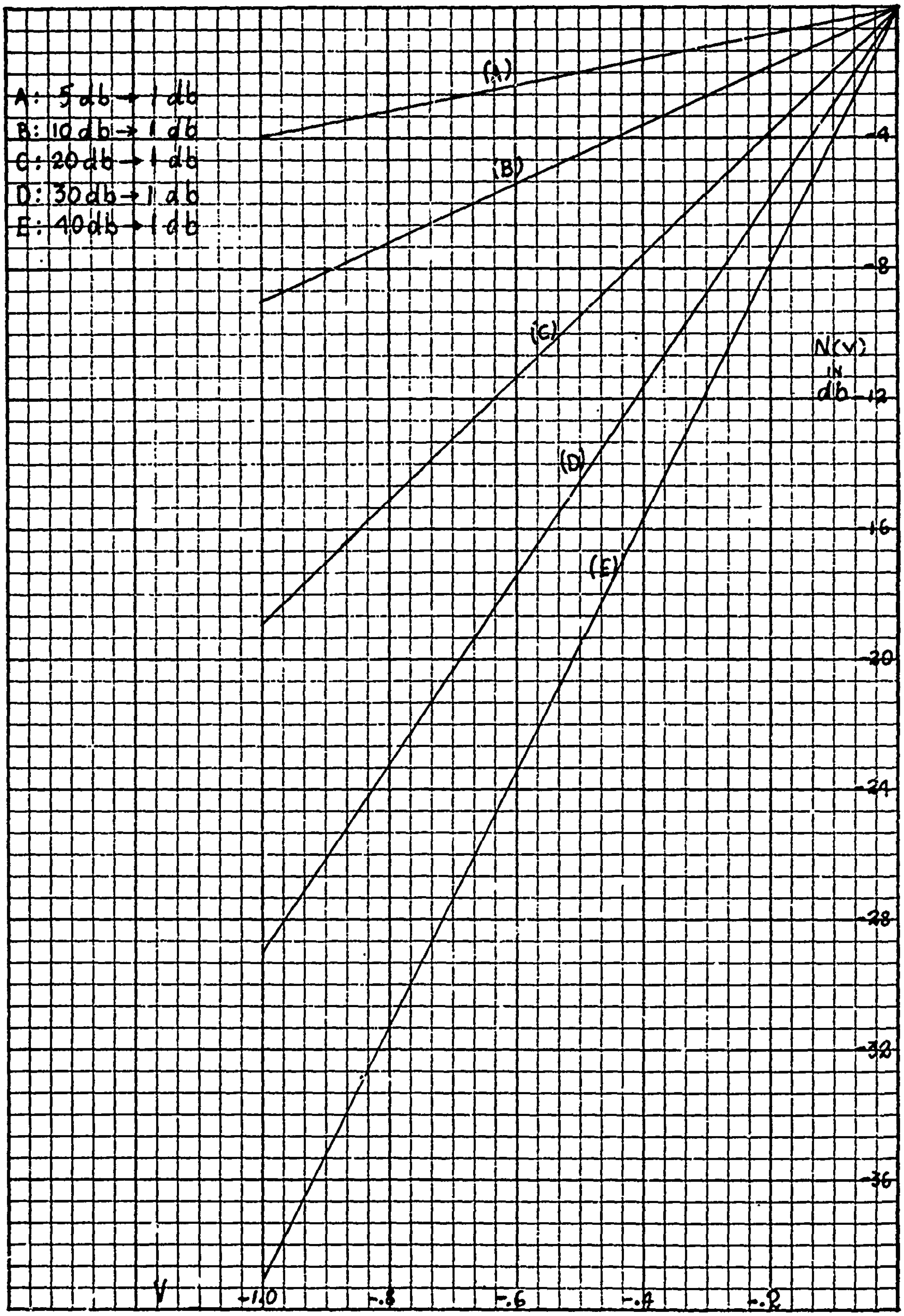


Figure 24. Linear Curves of Gain (in db) vs Bias Voltage.

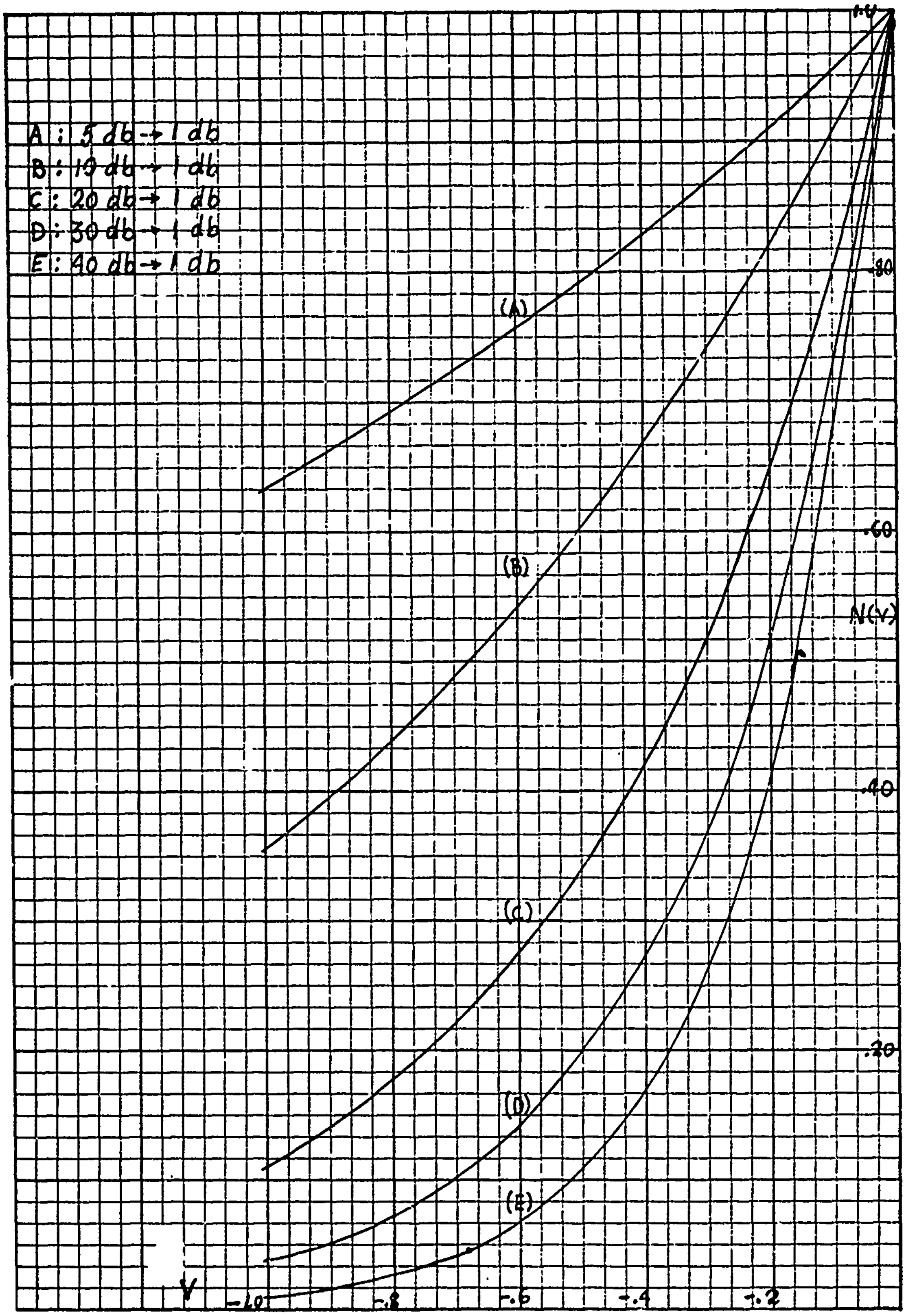


Figure 25. Exponential Plots of Gain vs Bias Voltage.

sensitive to random fluctuation of the bias voltage as was stated earlier.

An example of using the curves is worked out in Appendix F and it is proved that there exists a linear relationship between input and output amplitudes expressed in db.

The second case is believed to be of more importance because it preserves, under certain assumptions, the exact shape of the input waveform, thus eliminating distortion. In other words the output waveform is an exact copy of the input waveform but the amplitude is regulated between the prescribed bounds. The input-output characteristic of the whole loop is given in Fig. 26 and it is a straight line in linear coordinates.

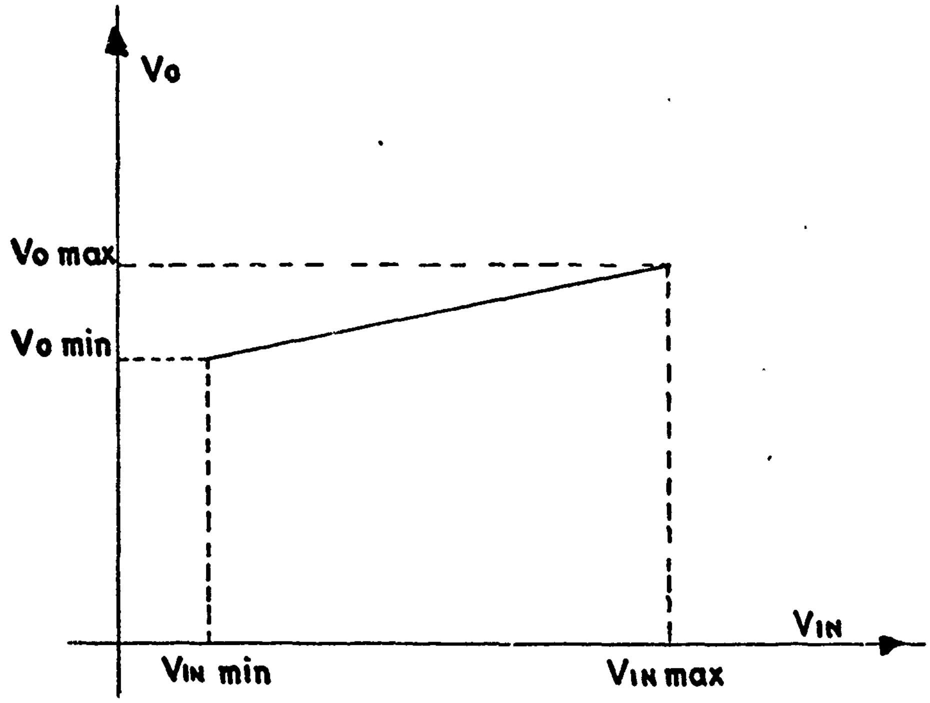


Figure 26. Input-output Characteristic of AGC loop maintaining Linearity Between Input and Output.

It is believed that this case is of importance because there is no reason why this should not be the ultimate goal of a designer if the application requires linearity between input and output and affords the extra effort and cost to implement the non-linear gain curve, the derivation of which is described below.

Solving equation (4-8) for the non-linear gain factor

$$N(v) = \frac{\overline{Vo}}{\overline{Vin}G_1} \tag{4-17}$$

assuming $G_1 = 1.0$ for convenience.

$$N(v) = \frac{\overline{Vo}}{\overline{Vin}} \tag{4-18}$$

Now taking points on the linear characteristic of Fig. 25 the corresponding quantities $\overline{Vo}$, $\overline{Vin}$ can be expressed

$$\overline{Vo} = Vomin + nA \tag{4-19}$$

$$\overline{Vin} = Vinmin + nB$$

where A and B are proportional increments in Vo and Vin correspondingly and $n = 0, 1,2,\ldots,N$.

Substituting into (4-18) the corresponding values of gain can be calculated as:

$$\lambda_n = \frac{Vomin + nA}{Vinmin + nB} \tag{4-21}$$

The values of gain determined in this manner can be plotted against arbitrary equidistant increments of bias voltage according to

$$v = nV \tag{4-22}$$

where V is the arbitrary increment of the bias voltage v.

Equations (4-21) and (4-22) are the parametric equations of the gain characteristic with parameter n. Eliminating the parameter between them

$$N(v) = \frac{VominV + vA}{VinminV + vB} \tag{4-23}$$

Assuming minimum values of input and output amplitude normalized to 1:

$$N(v) = \frac{V+vA}{V+vB} \tag{4-24}$$

which is the equation of the non-linear gain characteristic generating an output waveform of the same shape as the input waveform. Solving

$$v = \frac{(1-N(v))V}{N(v)B - A} \tag{4-25}$$

It is observed that equation (4-24) considering a negative bias voltage, is in a normalized form giving Nmax = 1.0 for v = 0.

One plot of equation (4-24) having parameters adjusted for maximum regulation 10:1 is shown in Fig. 27.

One can use the equation having assigned values for A and B for an excessive regulation; for example 10^5:1, and for applications requiring smaller regulation one small region of the graph is used.

For example an input having amplitude variations of 100 db ranges from 1 volt to 10^5 volts and the corresponding output amplitude extremes will be 1 volt to 1.12 volts.

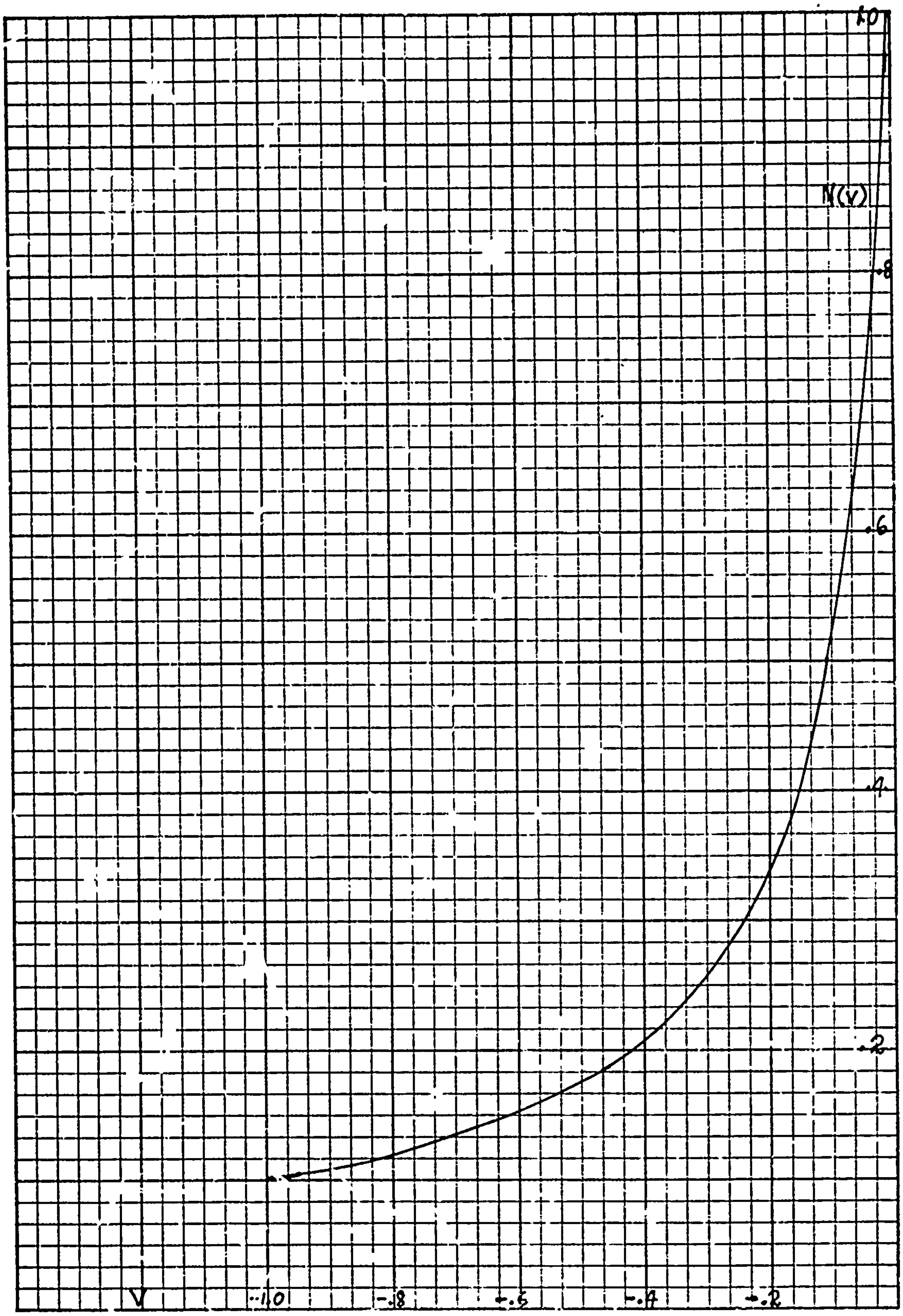


Figure 27. Plot of Equation (4-24) in Normalized Scales.

Then equation (4-24) becomes

$$N(v) = \frac{V+v}{V+10^5 v} \tag{4-26}$$

This particular equation has been used in Appendix F to regulate 20, 30 and 40 db input amplitude fluctuations.

The factor V as well as the remaining parameters around the loop have been assigned in accordance to the theory developed so far.

It is observed that equation (4-26) gives the two extreme values of gain at $v = 0$, $N_{max} = 1.0$ and $v \to \infty$, $N_{min} \to \frac{1}{10^5}$ thus the exact regulation of 101 db to 1 db is approached at the limit, but anything less than that can be theoretically achieved.

One last word of caution should be added here concerning the transient response of the system. During the transient phase of operation of the loop the output amplitude can take values well below and above the prescribed limits thus forcing the non-linear gain to values higher and lower correspondingly from the specified limits. It must also be kept in mind that at $t = 0+$ the bias voltage is zero so gain attains its maximum value.

Therefore designing an AGC loop, it will be a good practice to leave some margins below and above the operating region of the graph.

Now, the evaluation of the importance of equation (4-24) is as follows: Assuming the input waveform a pure sine wave the frequency of which is outside of the closed loop

frequency band the output will be for all practical purposes of the same shape, a pure sine wave, with peak amplitudes within the specified bounds of regulation.

If the input is an arbitrary waveform then it can be analyzed into sinusoidal Fourier components, and the output will be the summation of those components which are contained in the frequency band of the system, unregulated. This of course will produce a distortion to the output waveform.

If however the frequency band of the AGC system has been chosen arrow enough most of the Fourier components of input will be regulated and the remaining will be negligible in magnitude to affect the output waveform.

In Appendix F several simulations of AGC loops have been made using the two derived non-linear characteristics and proving that, for all practical purposes, they satisfy the conditions from which they were originated.

In example 2 the "distortionless" gain characteristic was tested and it yielded an almost excellent linearity between input and output for two different input waveforms.

In example 3 the phase shift produced by the AGC loop was observed and it was proved that it was a leading one as was expected.

In example 4 the transient response of the AGC loop using the "distortionless" characteristic is shown.

In example 5 an AGC loop having dynamics in the forward path and using the same as above gain characteristic was simulated and the results were still without noticeable distortion.

In example 6 the same system of example 5 was simulated and input waveforms of various frequencies were applied proving that for the frequencies outside the pass band of the loop the output is a replica waveform of the input with the output amplitude restricted between prescribed bounds. For frequencies inside the passband of the system the regulation is poor and the output waveforms are distorted.

V. CONCLUSIONS

Automatic gain control loops are non-linear systems and in some sense self-adaptive.

Classical feedback control theory is used throughout this work for the analysis of these systems containing an arbitrary number of dynamics.

The linearization procedure is the usual one in feedback control theory of assuming nominal levels for input and output and relating small perturbations of the input amplitude to resulting variations in the output amplitude.

Relationships among the various signals around the loop are established and formulas calculating basic features of the system are derived.

It is shown that the principal function of the AGC system, the regulation of the output level, is dependent on the loop gain.

The linear loop gain and the reference signal are specified according to steady-state requirements and specific worked examples verify the validity of the derived formulation.

A small signal transfer function is derived and its characteristic equation is found, as having a non-linear gain term which depends on the linear loop gain, the input signal amplitude and the slope of the characteristic of the variable gain amplifier at the operating region.

The characteristic equation specified as above is subjected to frequency response techniques and it is proved to be applicable and consistent in the prediction of the behavior of the system to specific input waveforms and in compensation of the system, to increase stability and achieve desirable transient and steady-state response.

The ideas and formulas derived in the analysis part are applied to the design of AGC loops with very good results. The specific features of the non-linear gain characteristic are discussed in connection with specific requirements in the transient and steady-state response of the loop. The results are used to formulate a general non-linear characteristic which under certain assumptions, stated in the text, eliminates the distortion associated with the non-linear nature of the AGC systems.

Examples are worked out, clearly showing all the main points in the analysis and in the synthesis part of the developed theory. All the worked examples are simulations of specific loops, the parameters of which are calculated, applying the formulas derived in the first four chapters of the thesis. For the simulation the "Continuous System Modeling Program" is used, available in the IBM/360 computer facility of the Naval Postgraduate School.

APPENDIX A

AGC LOOP GAIN

In this example an AGC loop was simulated using CSMP language, in order to verify that the loop gain depends upon the linear gain of the loop, the nominal amplitude of the input and the slope of the gain characteristic at the working region.

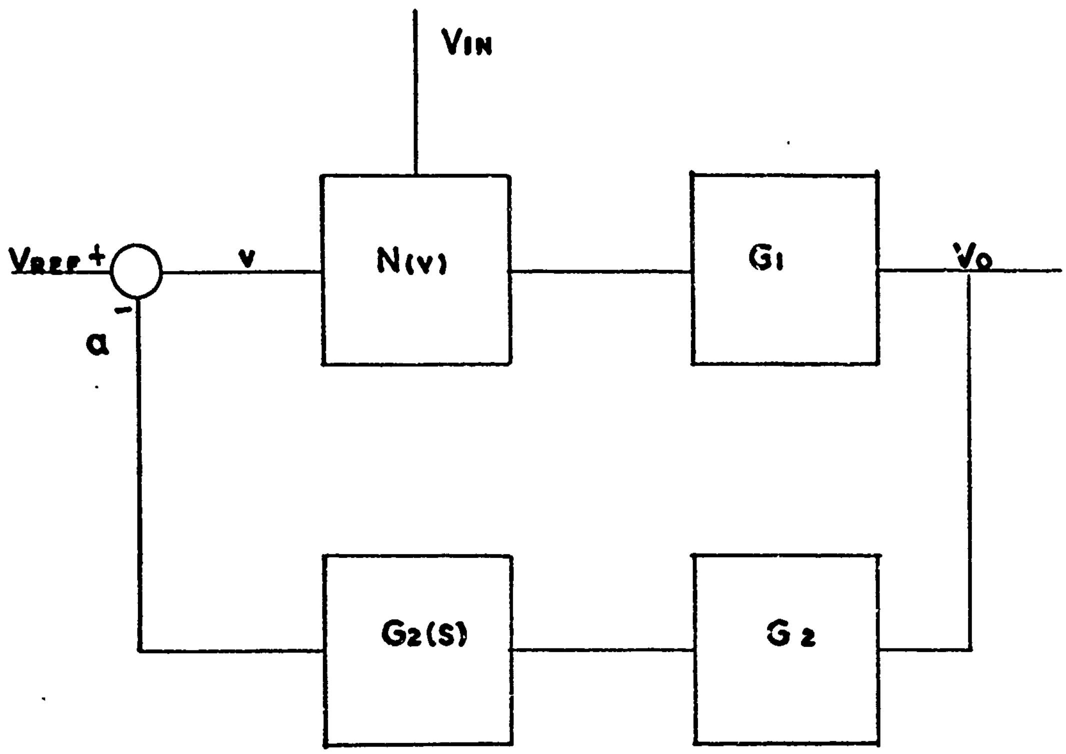


Figure 28. Block Diagram of the Simulated Example.

The characteristic of the amplifier gain was assumed linear, as is shown in Fig. 29 and $G_2(s) = \frac{P}{s+P}$ where $P = 100$.

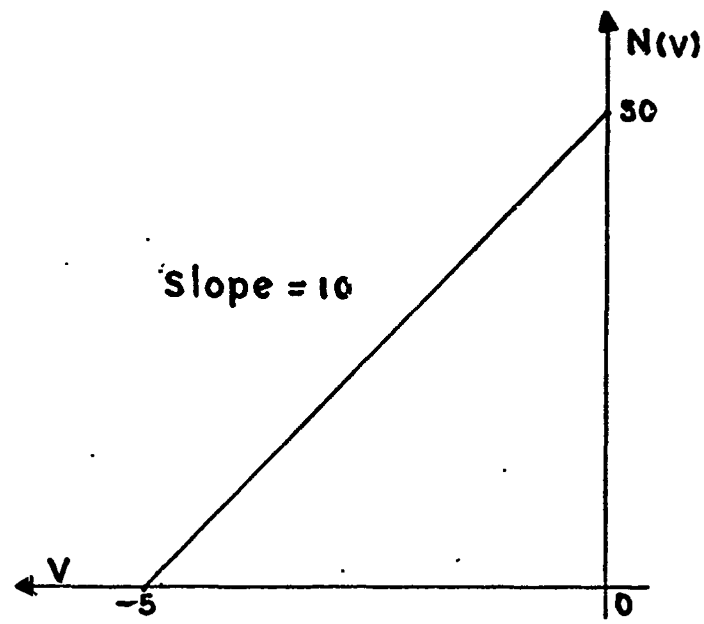


Figure 29. Gain Characteristic.

The successive simulations had the following form, with respect to the three factors which should be examined.

1. Vin = 1.25 + sin 50t

   Slope = 10 units/volt

   a. $G_1 = 1.0$, $G_2 = 1.0$

   b. $G_1 = 10.0$, $G_2 = 1.0$

   c. $G_1 = 1.0$, $G_2 = 10.0$

2. Vin = 1.25 + sin 50t

   $G_1 = 1.0$, $G_2 = 1.0$

   a. Slope = 5

   b. Slope = 10

   c. Slope = 15

3. $G_1 = 1.0$, $G_2 = 1.0$

Slope = 10

a. Vin = .125 + .1 sin 50t

b. Vin = 1.25 + sin 50t

c. Vin = 12.5 + 10 sin 50t

In all the cases the input amplitude had fluctuations of 20 db. The influence of the particular factor on the loop gain becomes clear observing the variations in the regulation of the output amplitude.

So the following ranges of output amplitude in db were observed:

1. a. 3.12 db

   b. .4 db

   c. .4 db

2. a. 5.16 db

   b. 3.12 db

   c. 2.24 db

3. a. 8.6 db (not taking into account the peaks)

   b. .5 db

   c. Very small, kind of noise effect due to integration method.

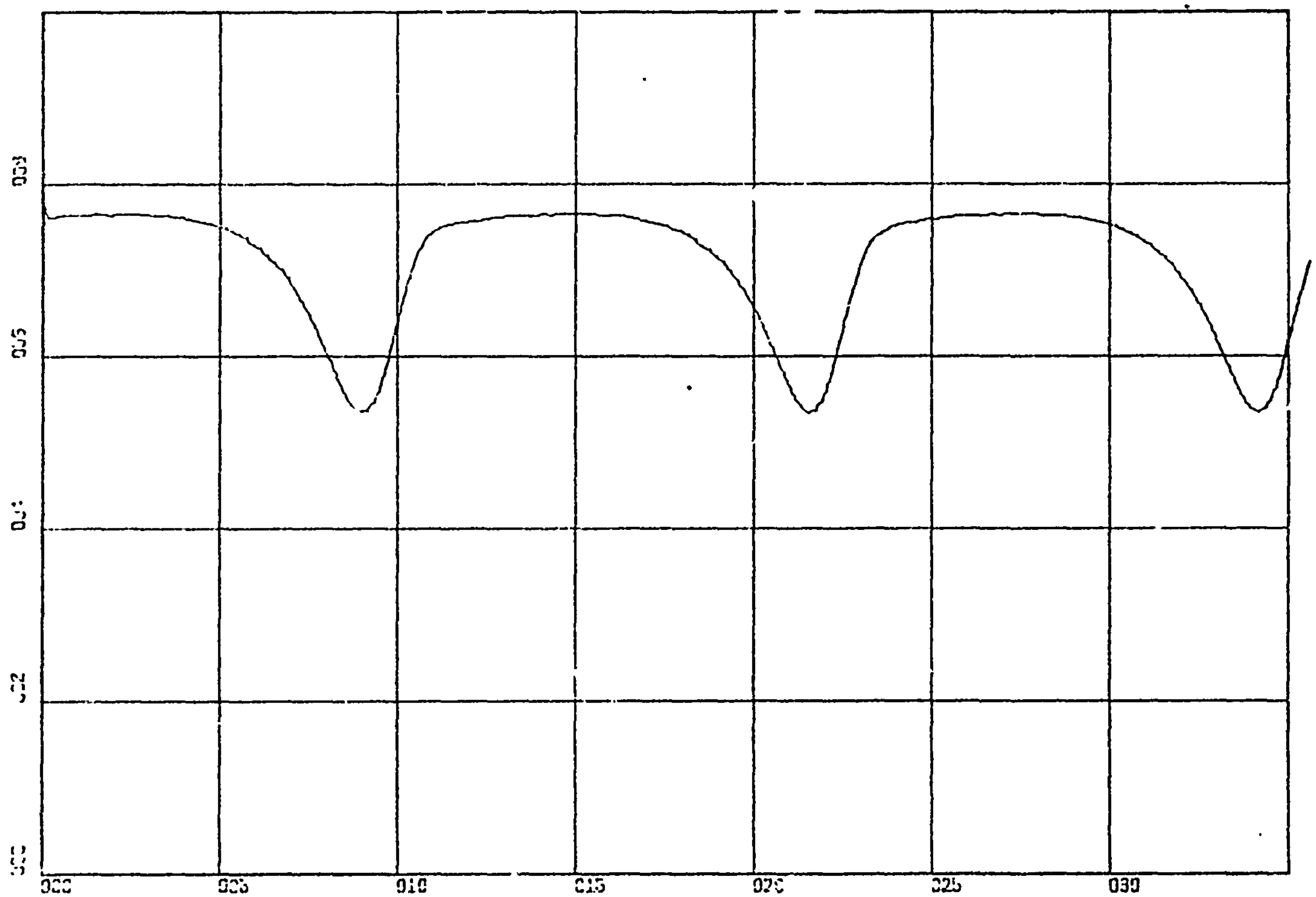
The following graphs are the plots of output amplitude vs time in which the influence of each factor is shown.

One result which should be expected from the developed theory is the cases 1b and 1c.

In both cases the total linear gain of the loop was

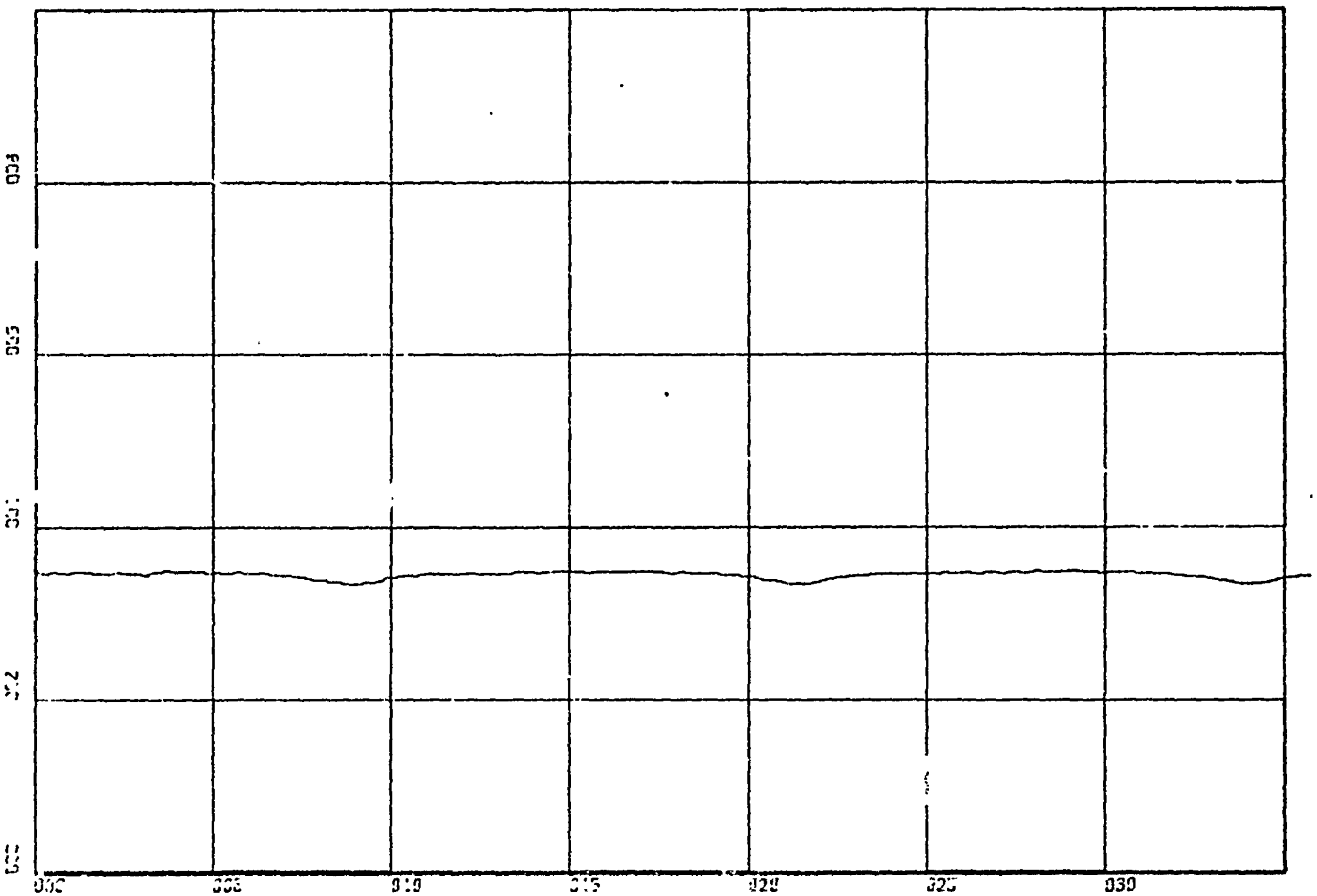
10.0 so the expected regulation (keeping the other factors the same) was the same.

The following plots for the two cases do not show clearly the effect because the level of the output value has been increased and it seems that case 1b obtains better regulation than 1c. If however one considers the ratio $\frac{Vomax}{Vomin}$ it is the same for both cases 1.045 or .4 db.



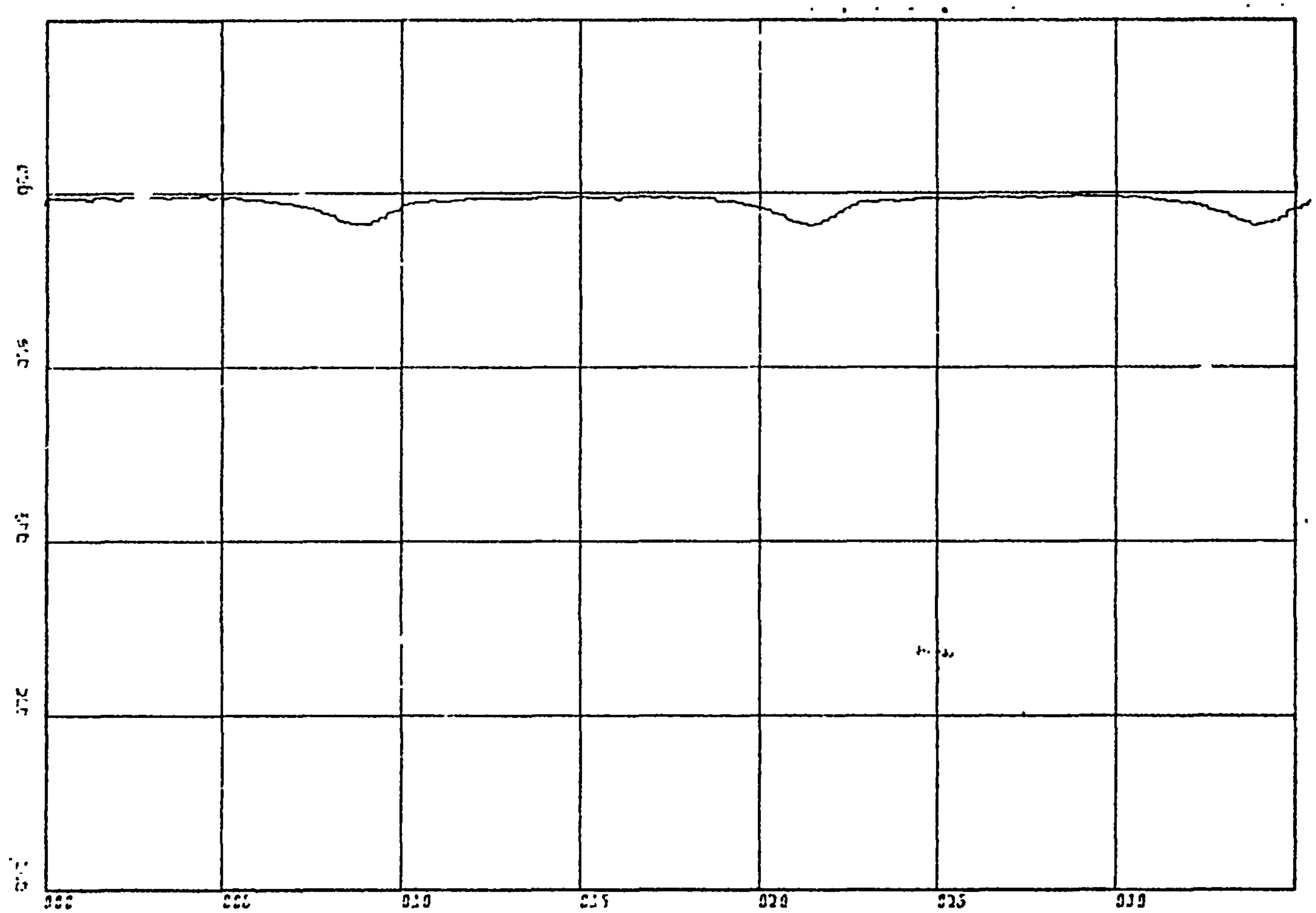
X: .05, Y: 2.0

Figure 30. Plot of Vo, $G_1 = 1.0$, $G_2 = 1.0$.



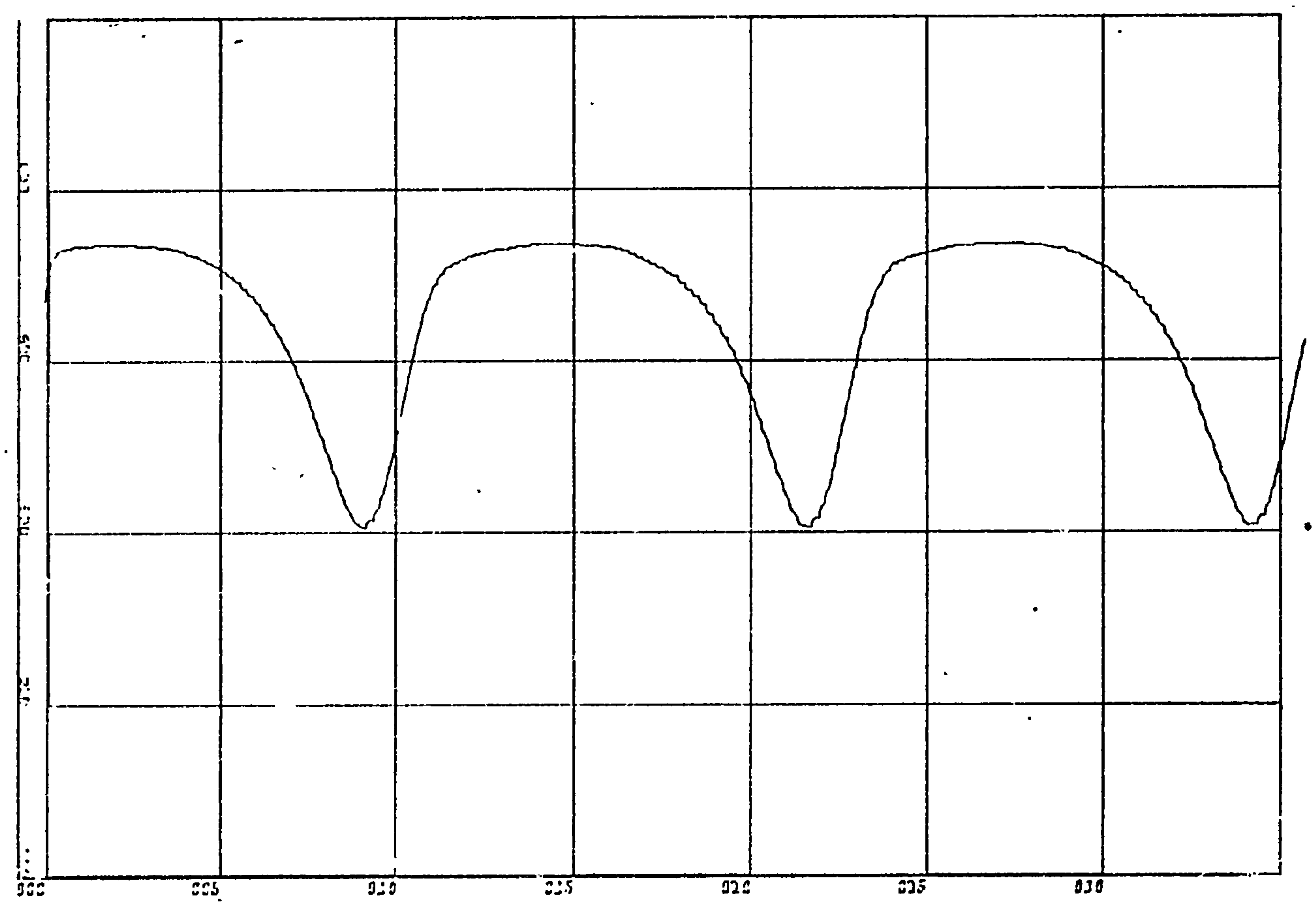
X: .05, Y: 2.0

Figure 31. Plot of Vo, $G_1 = 1.0$, $G_2 = 10.0$.



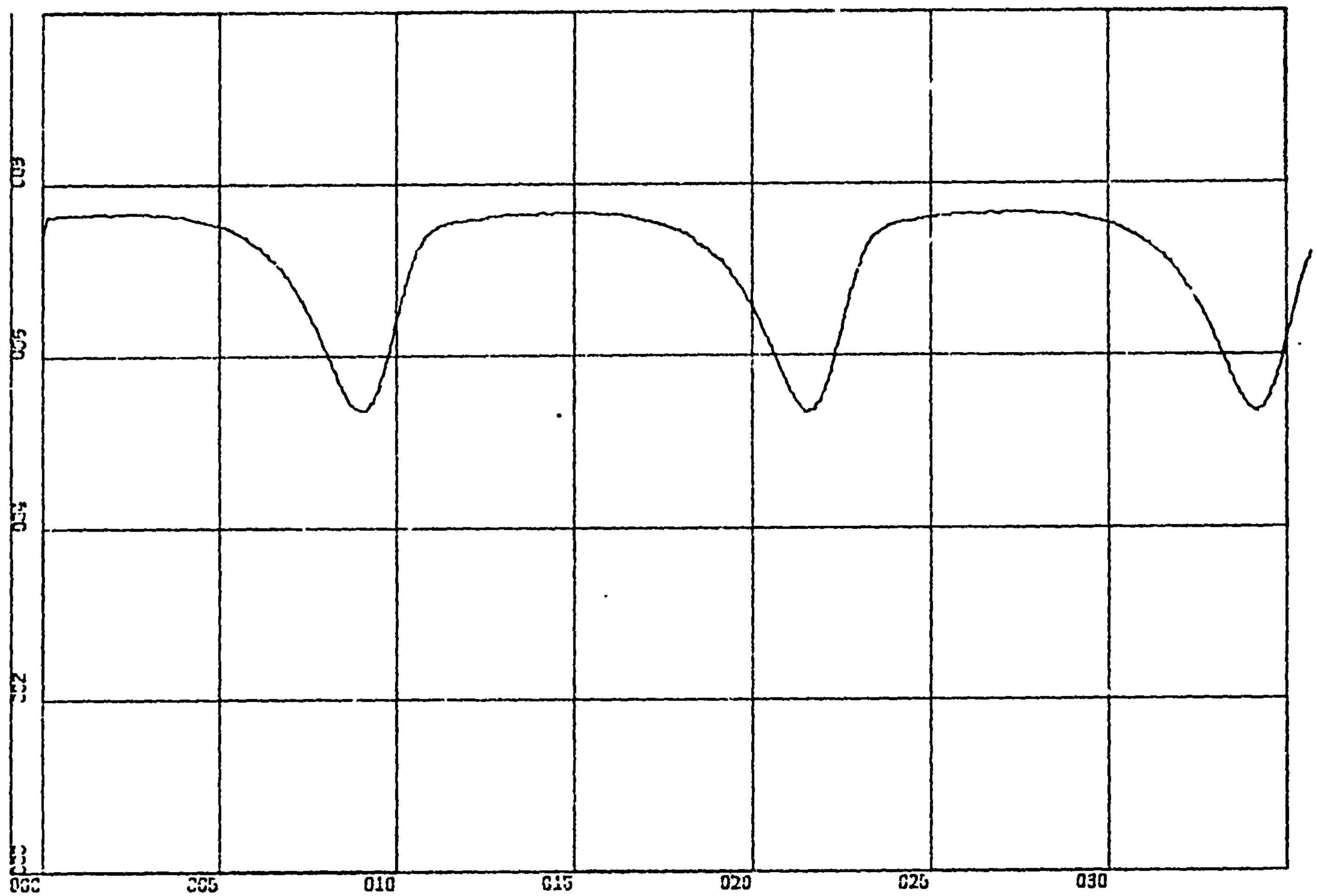
X: .05, Y: 2.0

Figure 32. Plot of Vo, $G_1 = 10.0$, $G_2 = 1.0$.



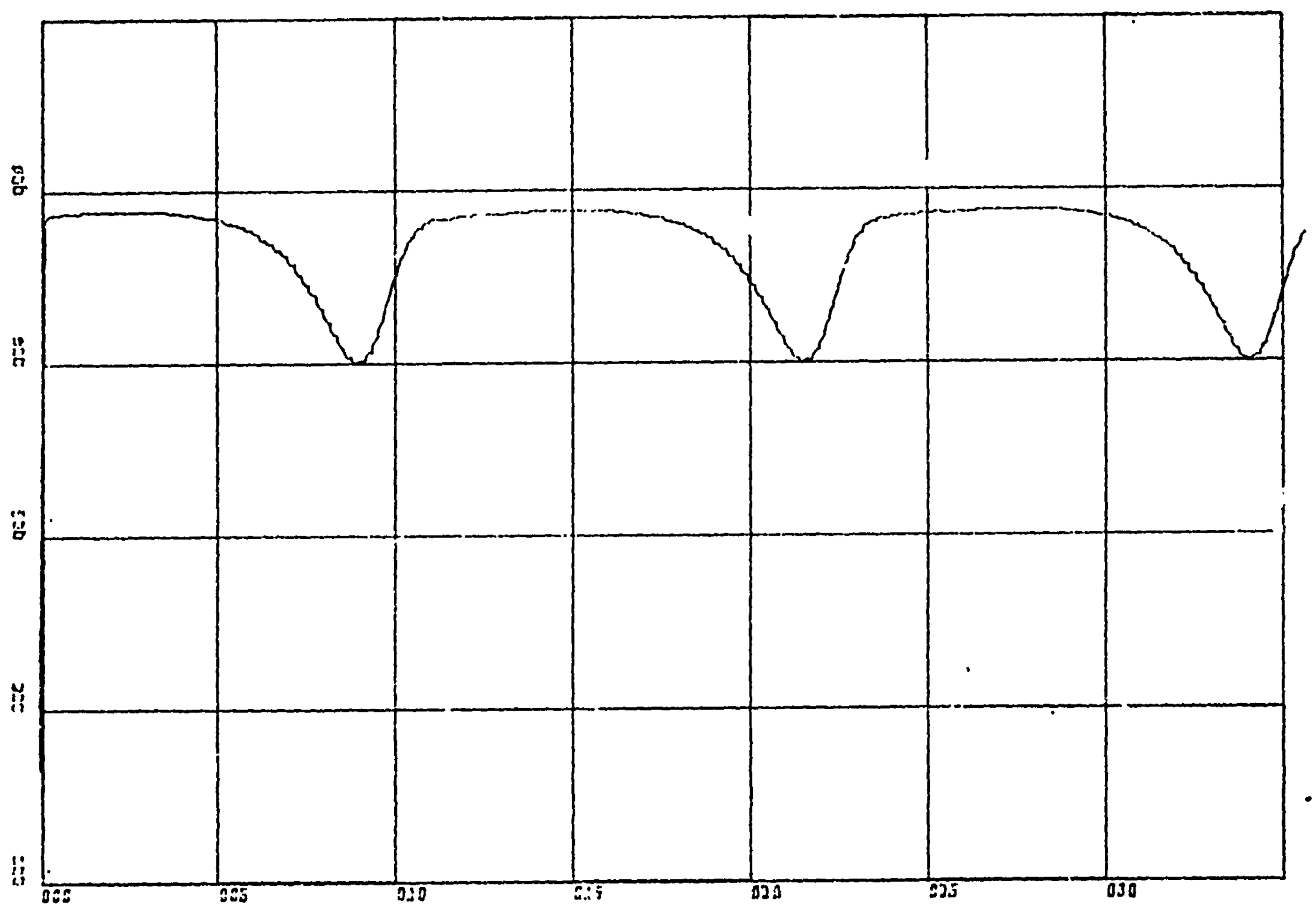
X: .05, Y: 2.0

Figure 33. Plot of Vo, Slope = 5.0.



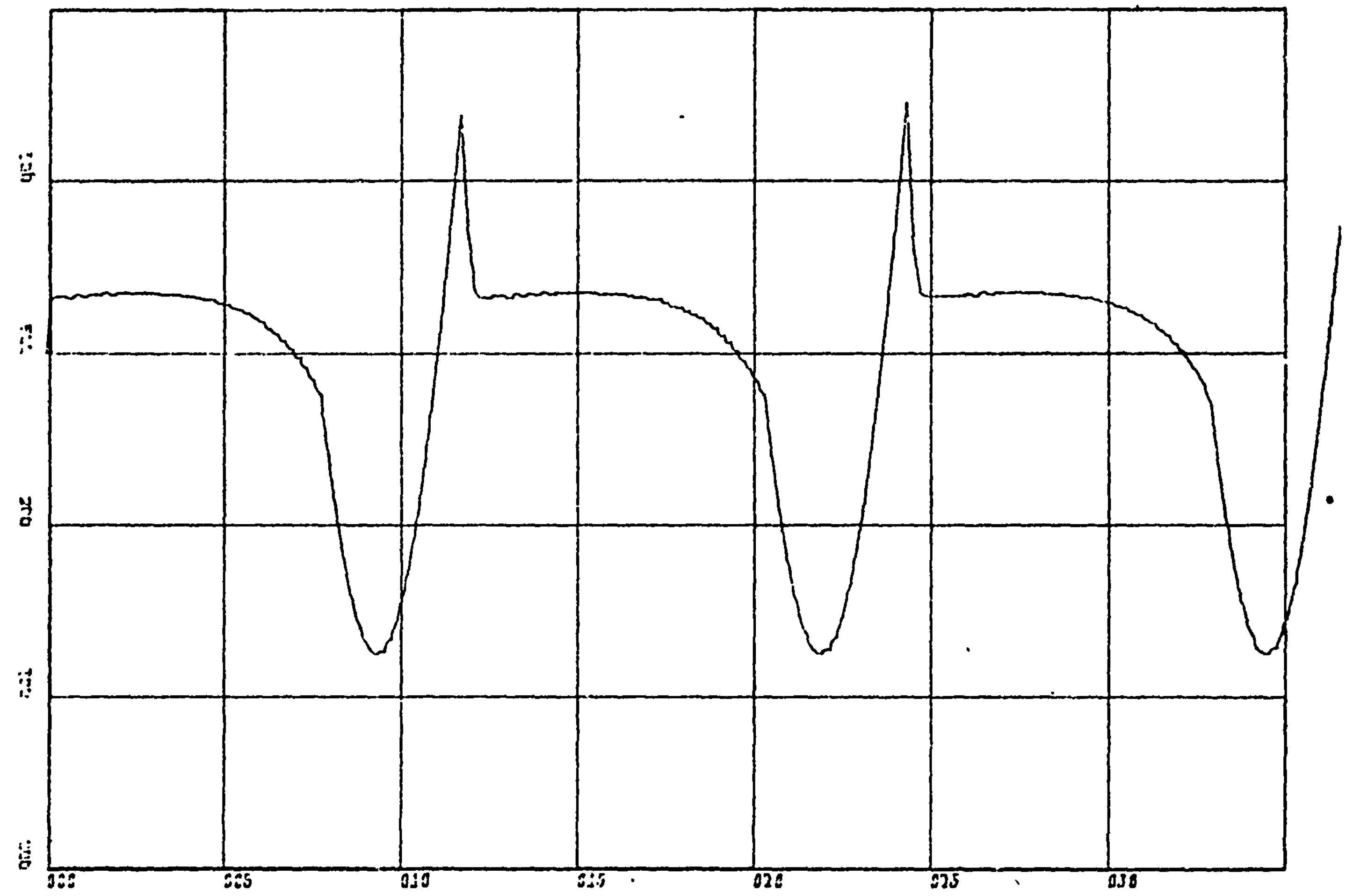
X: .05, Y: 2.0

Figure 34. Plot of Vo, Slope = 10.0.



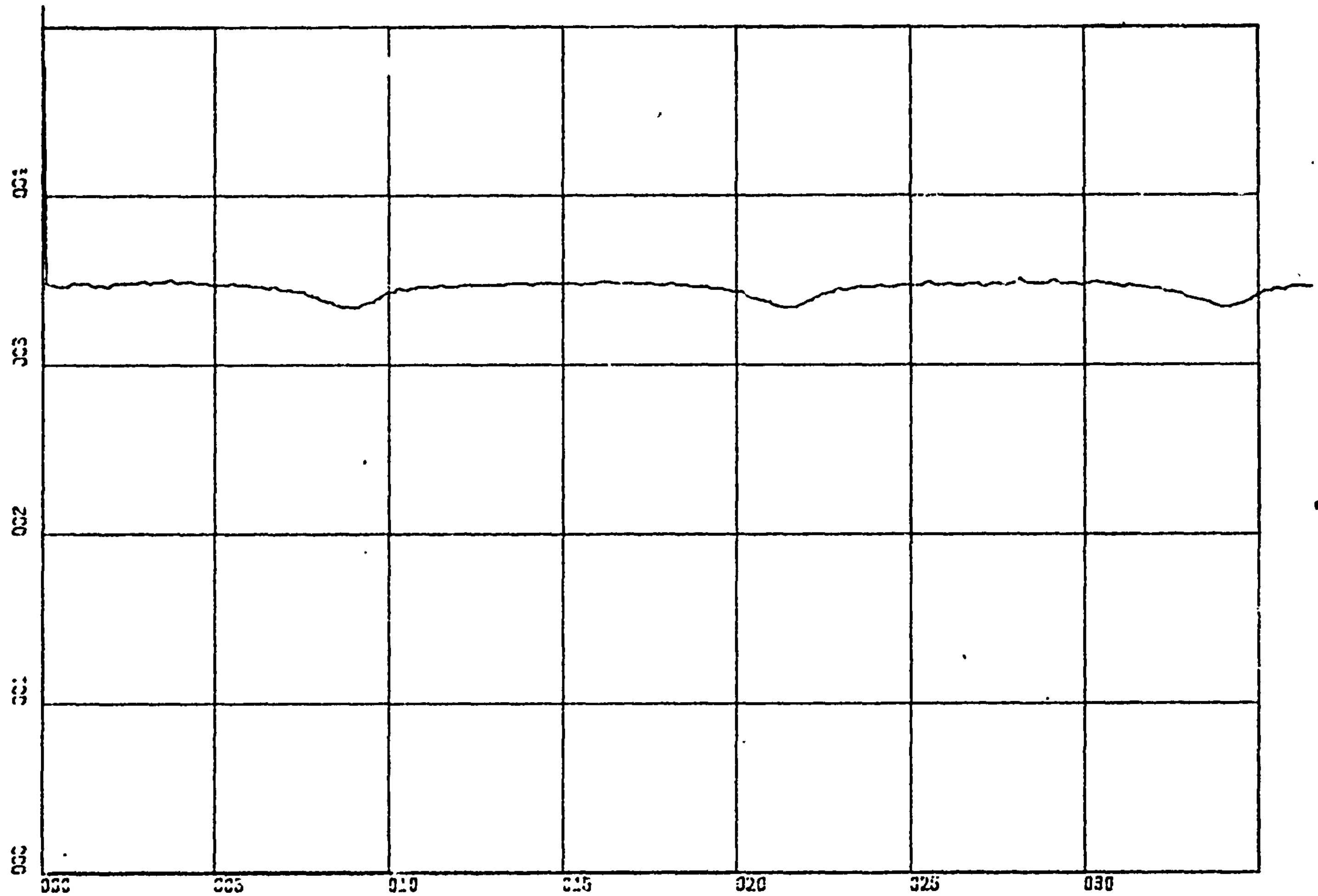
X: .05, Y: 2.0

Figure 35. Plot of Vo, Slope = 15.0.



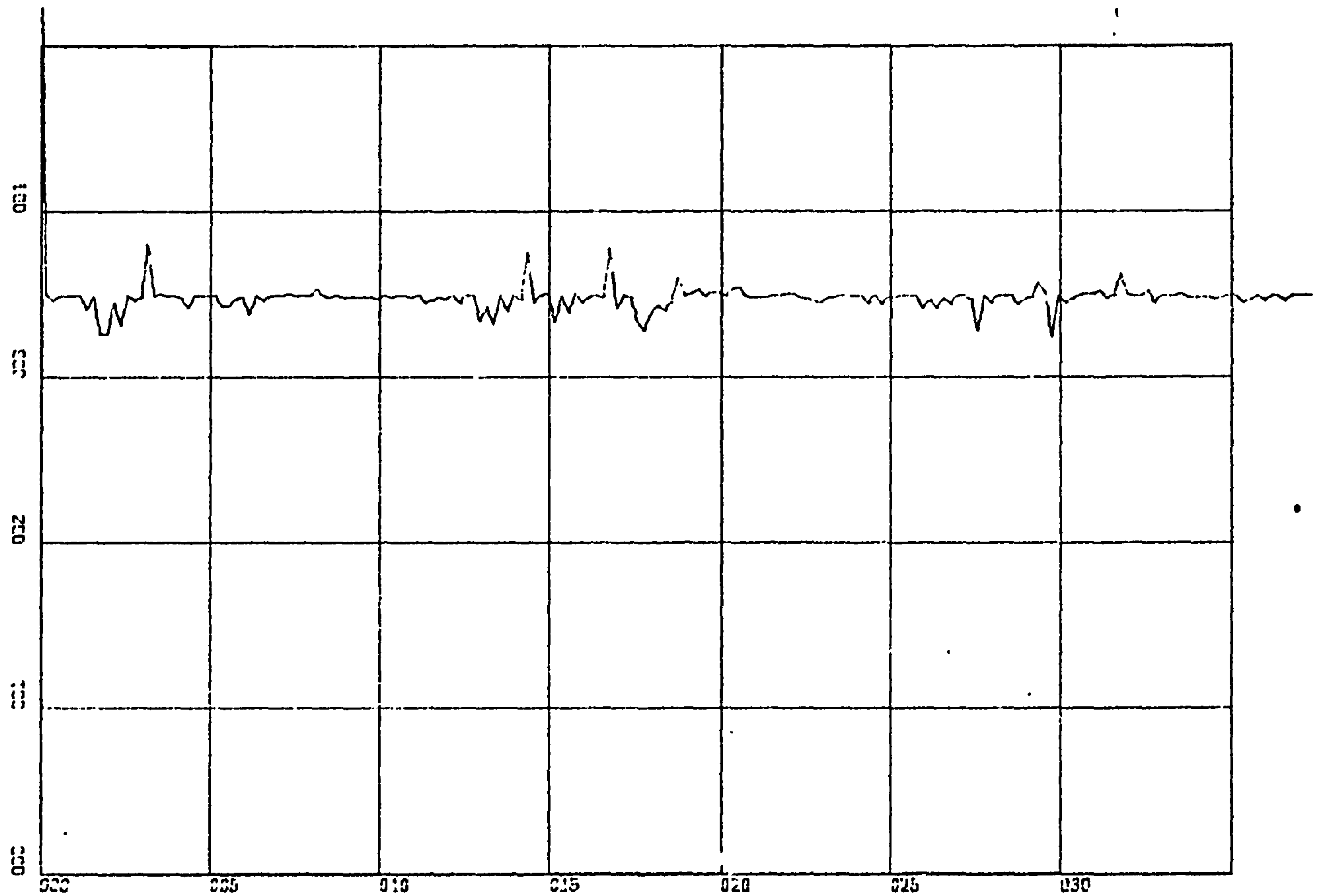
X: .05, Y: 1.0

Figure 36. Plot of Vo, Vin = .125 + .1 sin 50t.



X: .05, Y: 1.0

Figure 37. Plot of Vo, Vin = 1.25 + sin 50t.



X: .05, Y: 1.0

Figure 38. Plot of Vo, Vin = 12.5 + 10.0 sin 50t.

APPENDIX B

STABILITY

In this example a system was posed having the block diagram representation shown in Fig. 39.

According to the theory developed in Section III-A the characteristic equation of this loop was

$$1 + G \frac{p_1 p_2 p_3}{(s+p_1)(s+p_2)(s+p_3)} \tag{B-1}$$

where G was the loop gain and it was given by

$$G = G_1 G_2 \overline{Vin} \text{ (slope)} \tag{B-2}$$

The non-linear gain N(v) was assumed to be

$$N(v) = 20v + 100 \tag{B-3}$$

therefore the slope was 20.

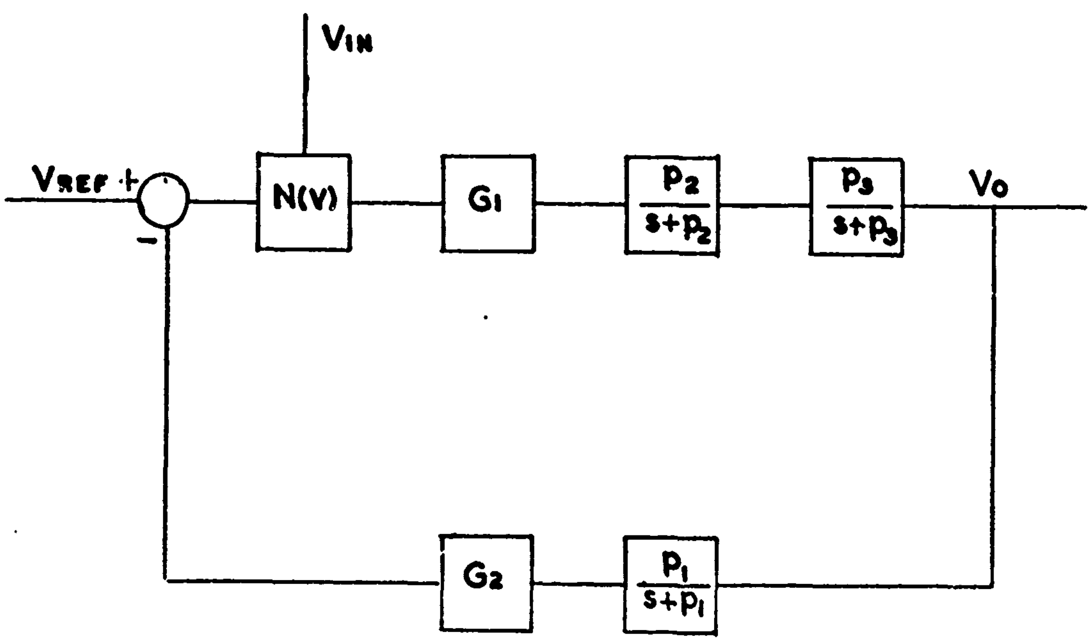


Figure 39. Block Diagram of the Simulated Example.

The following values were assigned to the various quantities around the loop.

$p_1 = 100.0$

$p_2 = 2000.0$

$p_3 = 2000.0$

$G_1 = 1.0$

$G_2 = 20.0$

First a constant input amplitude $\overline{Vin} = .05$ was assumed so the loop gain became

$$G = 1.0 \times 20.0 \times .05 \times 20.0 = 20.0$$

Having this loop gain as reference a BODE plot of the characteristic equation was drawn which is the one in Fig.40.

As the input amplitude is increased the loop gain is increased and therefore the magnitude curve on the BODE plot is raised vertically up to the point when both the magnitude and phase curve cross the 0 db axis at point A and the system becomes unstable.

The gain margin is 7 db therefore the input amplitude may be increased by a factor 2.24 (i.e. $\overline{Vin} = .11$) before the system becomes unstable.

The same results were obtained by applying Routh's criterion on the characteristic equation and checking with the root locus plot of the system.

The characteristic equation of the system is

$$(s+100)(s+2000)^2 + K = 0 \qquad \text{(B-4)}$$

or

$$s^3 + 4100\,s^2 + 44 \times 10^5 s + 4 \times 10^8 + K = 0$$

Routh's criterion

$$\begin{array}{c|ll}
s^3 & 1 & 44 \times 10^5 \\
s^2 & 4100 & 4 \times 10^8 + K \\
s^1 & \dfrac{18 \times 10^9 - 4 \times 10^8 - K}{4100} & \\
s^0 & 4 \times 10^8 + K &
\end{array} \qquad K < 17.6 \times 10^9$$

But

$$K = p_1 p_2 p_3 G_1 G_2 \overline{Vin} \cdot \text{Slope} = 100 \cdot 2000^2 \times 20 \times 20 \times \overline{Vin}$$

$$= 16 \times 10^{10}\ \overline{Vin}$$

Therefore

$$16 \times 10^{10}\ \overline{Vin} < 17.6 \times 10^9$$

or

$$\overline{Vin} < .11$$

which is exactly the same result found from the BODE plot.

On the other hand:

$$4100 s^2 + 4 \times 10^8 + 16 \times 10^{10} \times .11 = 0$$

or

$$s^2 = -4.3 \times 10^6$$

and

$$\omega = 2070$$

is the radian frequency at which the root locus crosses the imaginary axis, which is the same as the frequency of phase crossover of the BODE plot.

The root locus of the system is shown in Fig. 41.

Finally for inputs of smaller amplitude than .11 the system was expected to be stable and for inputs of larger amplitude than .11 the system was expected to be unstable, producing a limit cycle of frequency 2,070 rad/sec.

In the simulation attempted step inputs .01, .05, .08, .11, .15 were applied. Although this is not the conventional use of AGC systems, however it is a convenient way to show the application of the developed ideas about the stability of the loop.

The derived curves clearly indicate the change in the stability of the system as the loop-gain increases by means of increasing the input amplitude. For the last two cases the system becomes unstable and a limit cycle is produced the frequency of which was found ω = 2093 rad/sec in accord with the prediction above, and the amplitude of the limit cycle depends upon the loop gain being larger for $\overline{Vin}$ = .15 and smaller for $\overline{Vin}$ = .11.

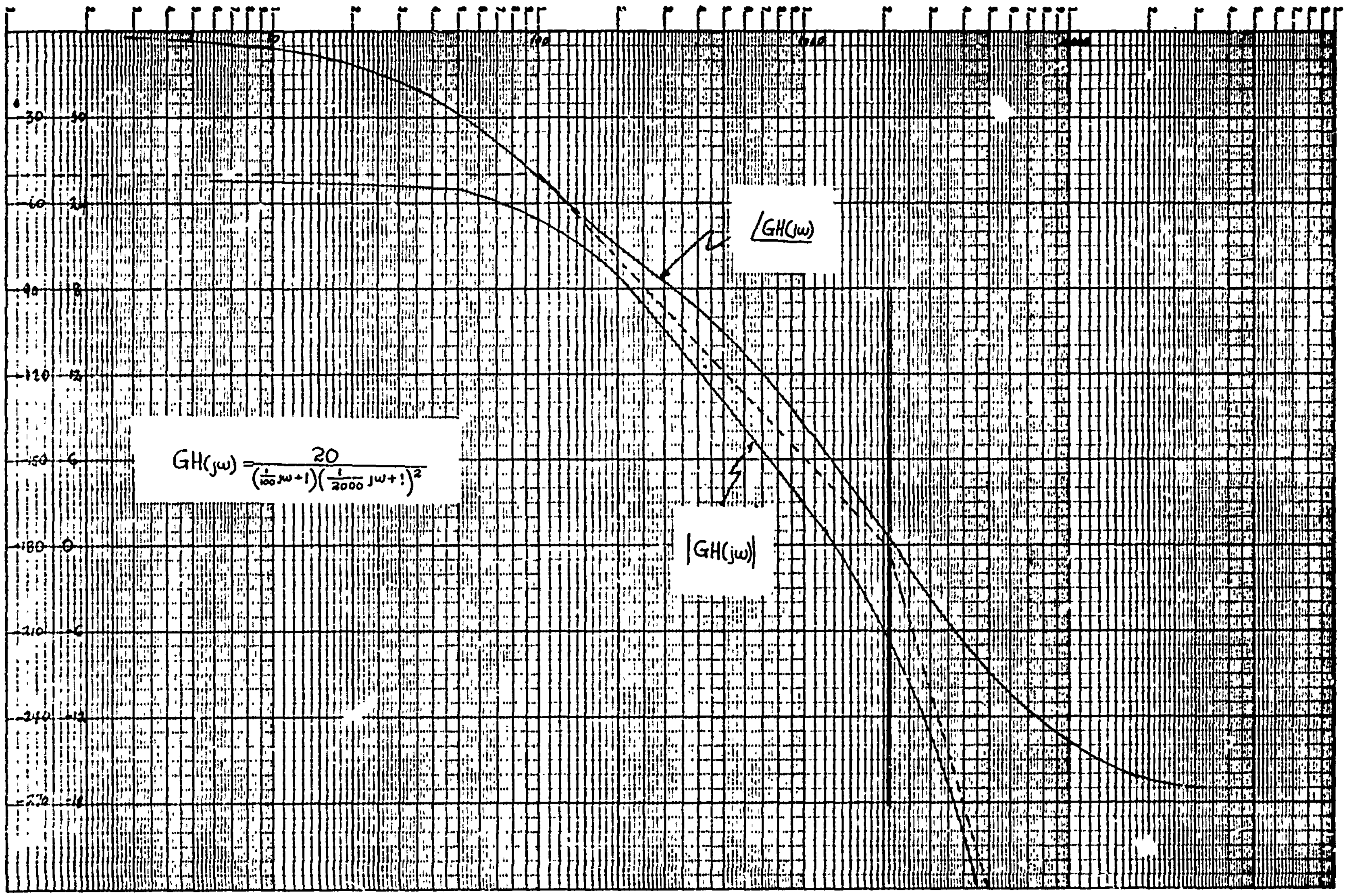


Figure 40. BODE Plot of Example No. 2.

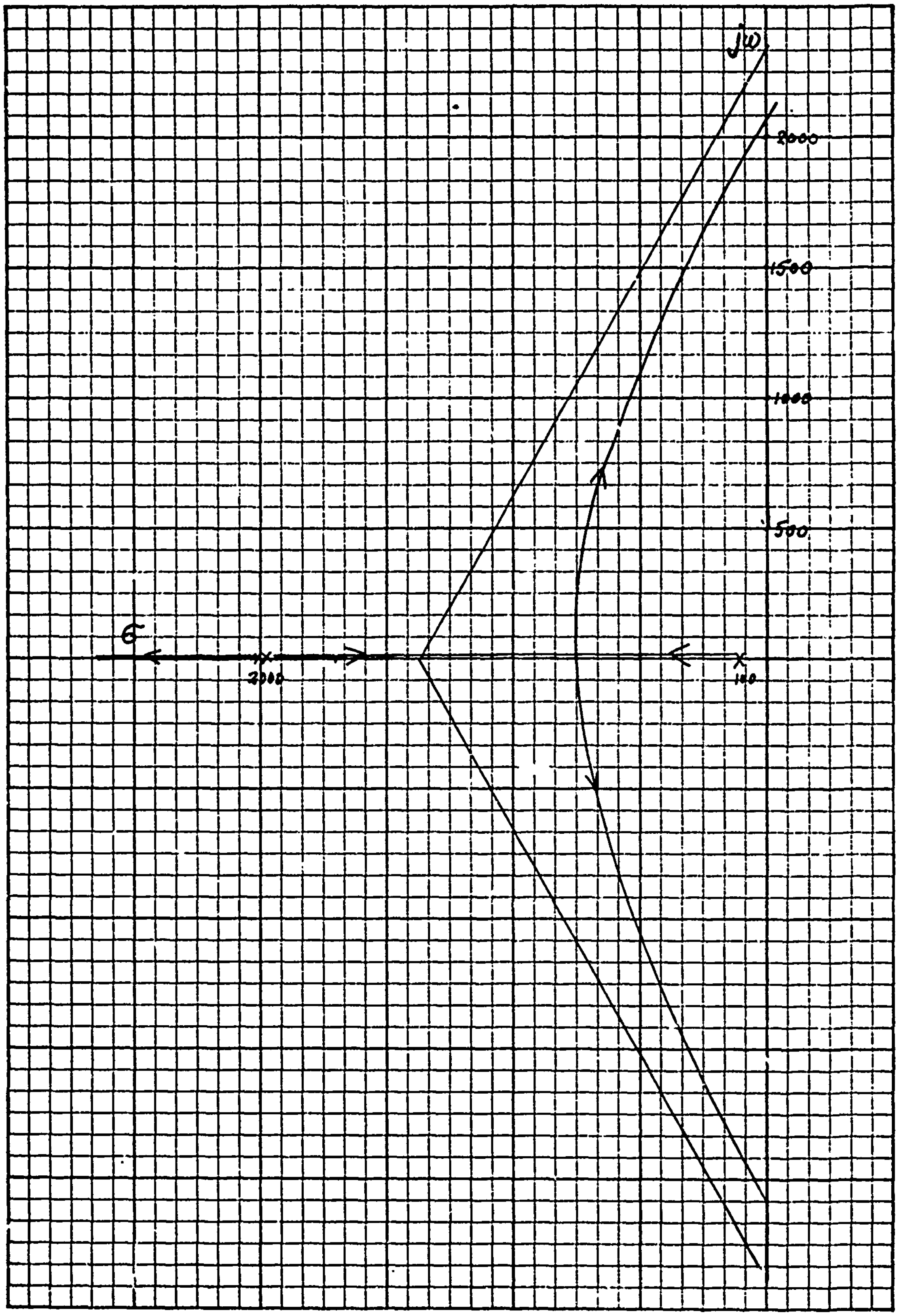
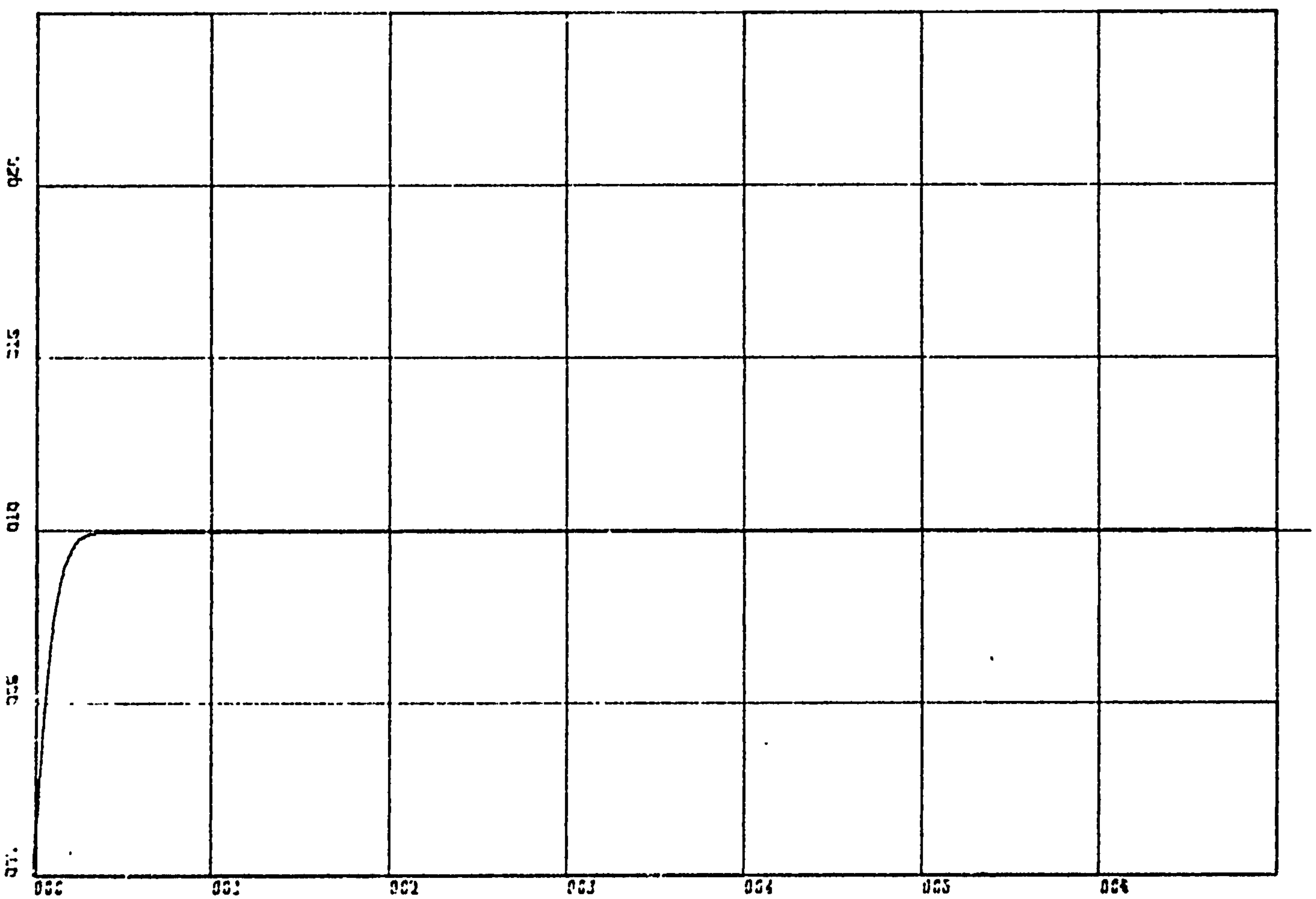
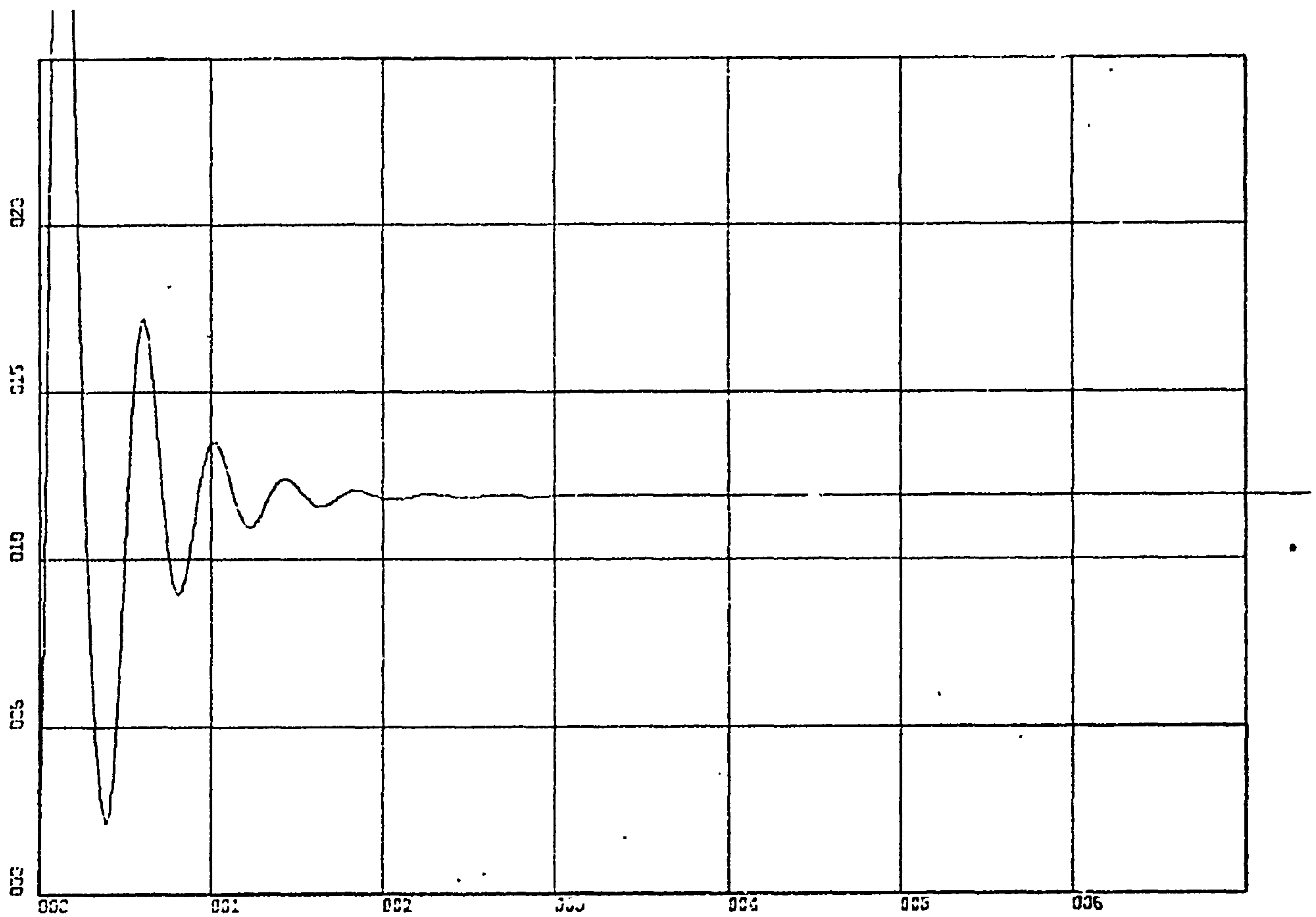


Figure 41. Root Locus Plot of Example No. 2.



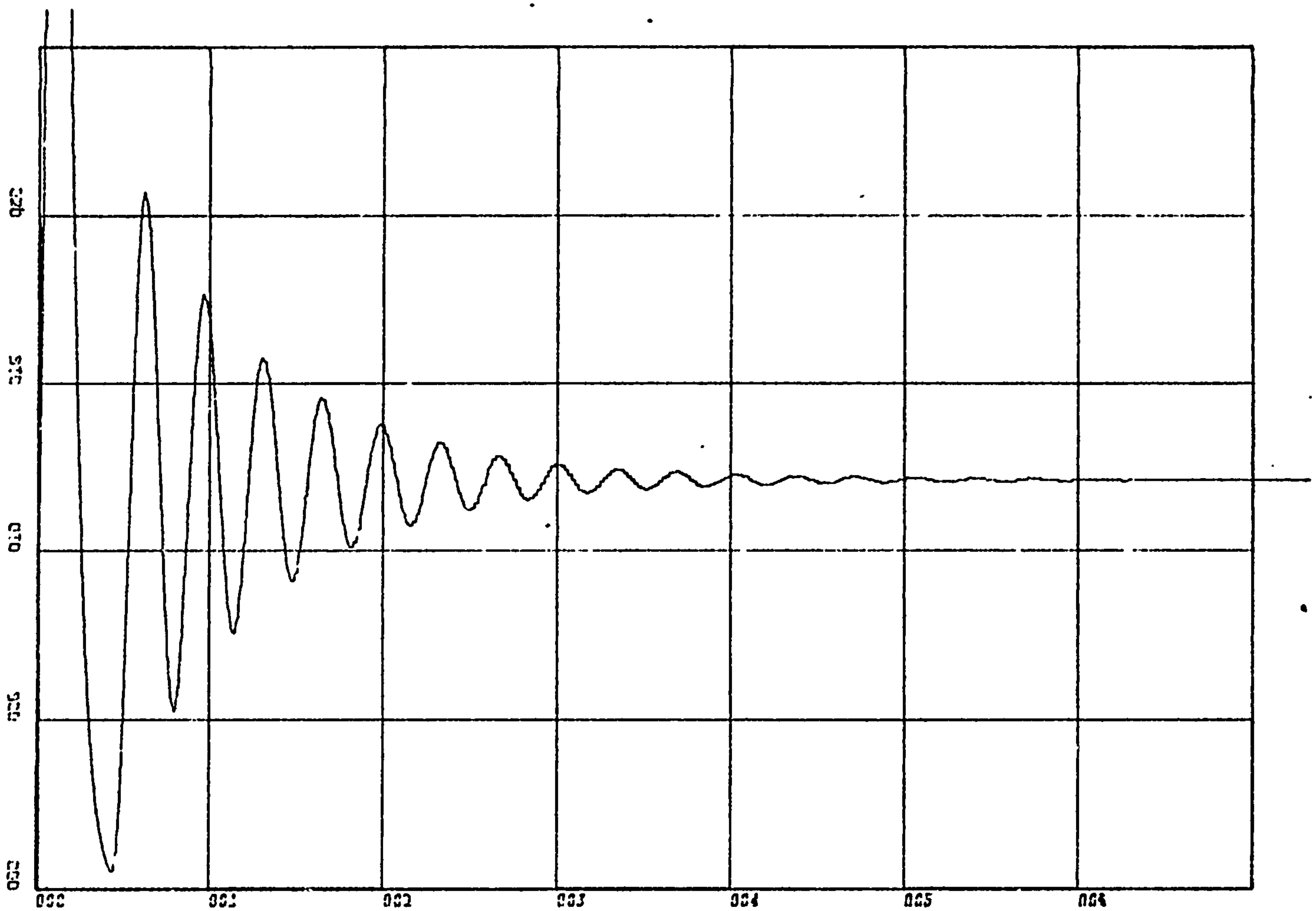
X: .01, Y: .5

Figure 42. Plot of Vo, Vin = 0.01.



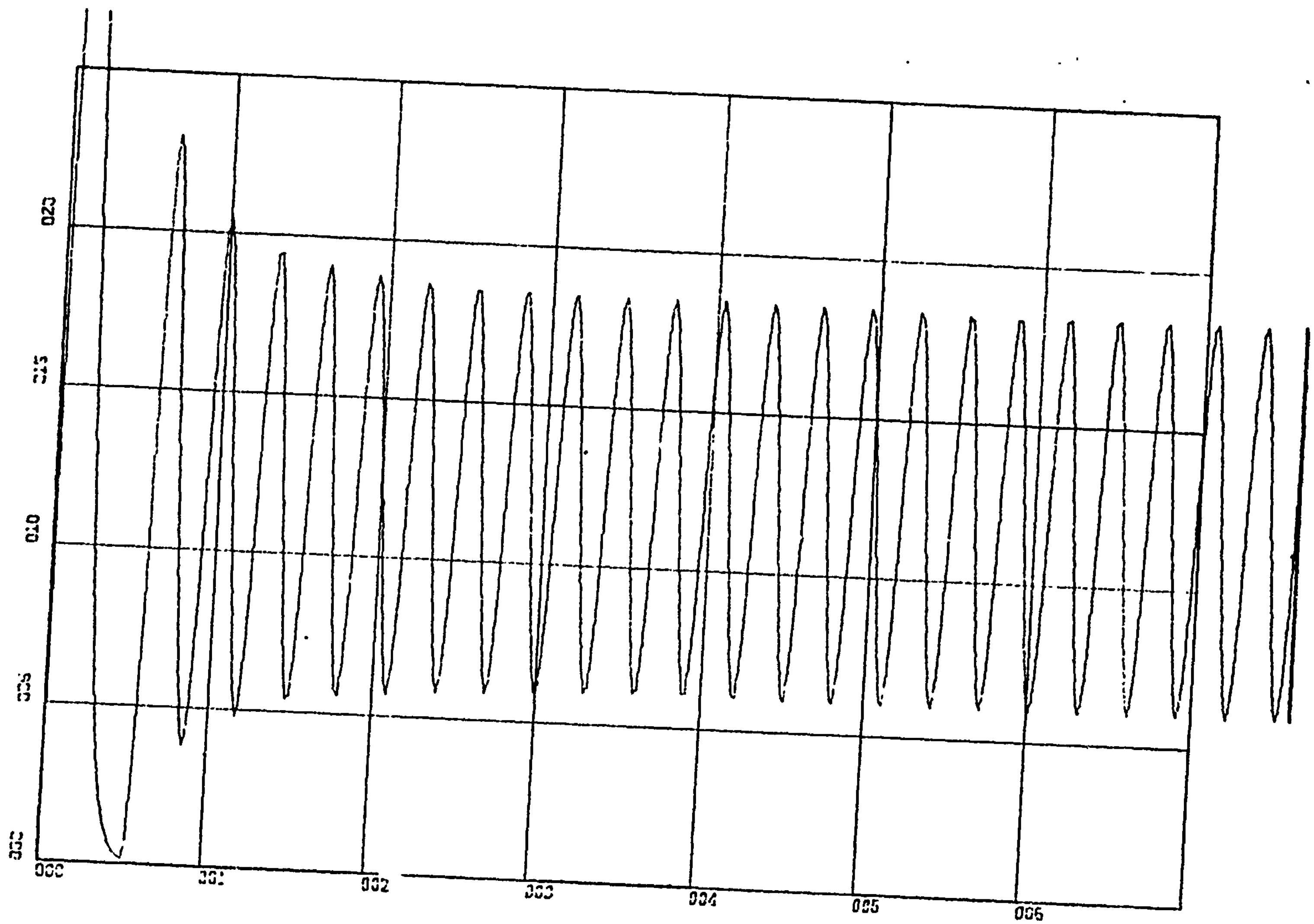
X: .01, Y: .5

Figure 43. Plot of Vo, Vin = 0.05.



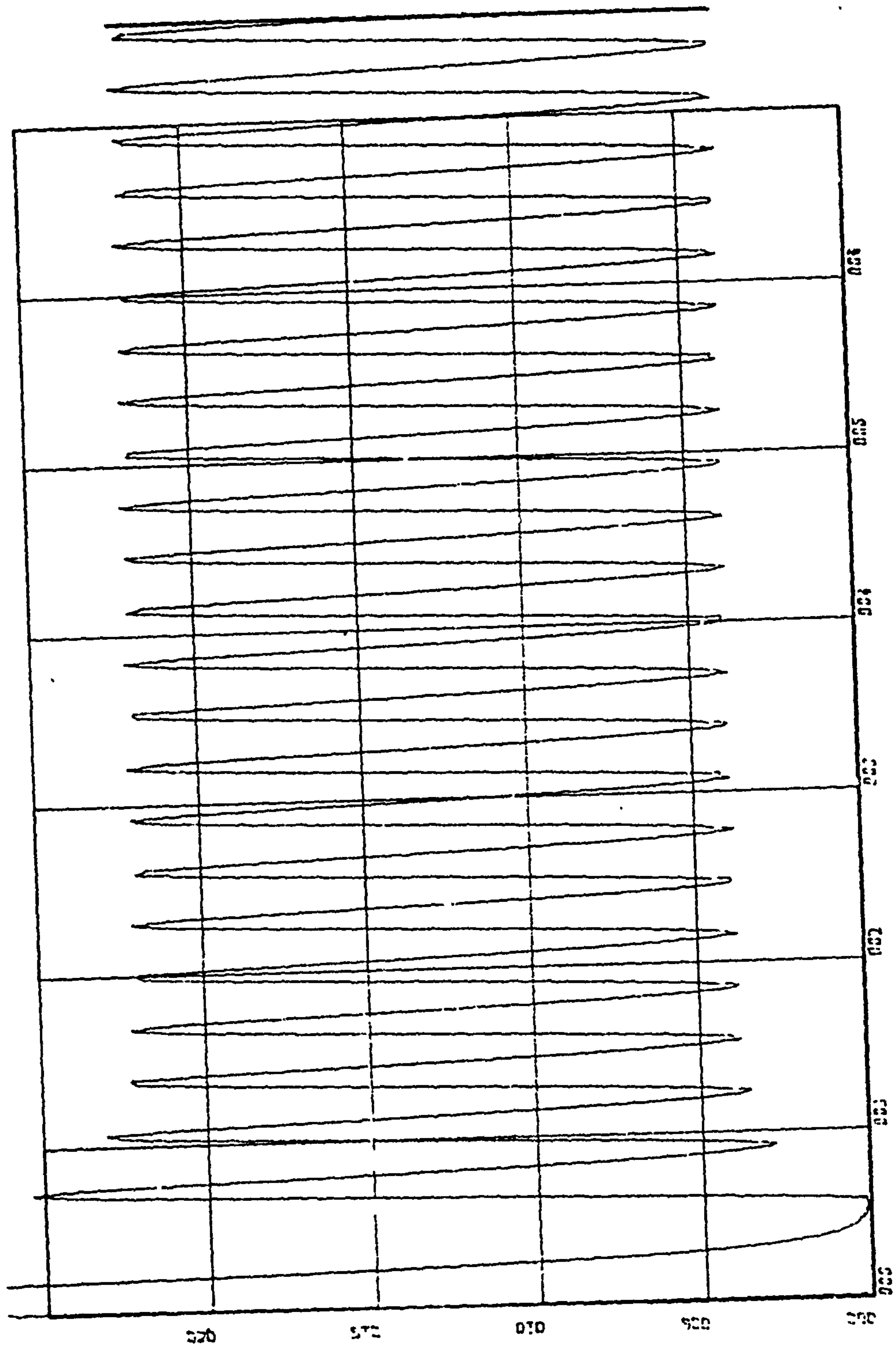
X: .01, Y: .5

Figure 44. Plot of Vo, Cin = 0.08.



X: .01, Y: .5

Figure 45. Plot of Vo, Vin = 0.11.



X: .01, Y: .5

Figure 46. Plot of Vo, Vin = 0.15.

APPENDIX C

EFFECT OF REFERENCE LEVEL ON AGC ACTION

Suppose the AGC loop block diagram as given in Appendix A, Fig. 28. Suppose also the non-linear gain characteristic is the one shown in Fig. 48.

The values of $G_1 = 1$ and $p = 100$.

Simulations were made for

a. $G_2 = 1.0$ $\quad V_{REF} = 0$

b. $G_2 = 1.0$ $\quad V_{REF} = 10.0$

c. $G_2 = 10.0$ $\quad V_{REF} = 10.0$

The inpur signal was always assumed to be a ramp function of slope 1.

Figures 49, 50 and 51 show the output voltage Vo plotted against time for the three cases.

It is seen that when $V_{REF} = 0$ as in Fig. 49 AGC action starts as soon as output appears out of the loop, compared with Fig. 50 in which regulation occurs when the output voltage Vo exceeds the level $\frac{V_{REF}}{G_2} = 10.0$

Fig. 51 shows also regulating action only for values of Vo exceeding $\frac{V_{REF}}{G_2} = 1.0$. The hump of the curve is due to the response of the feedback RC filter when a ramp is imposed at its input; Fig. 47 shows how this is generated. In the same graph the input to RC filter Vo and the output "a" have been plotted. In order for AGC action to start, "a" has to reach the level $\frac{V_{REF}}{G_2}$, but by that time Vo has

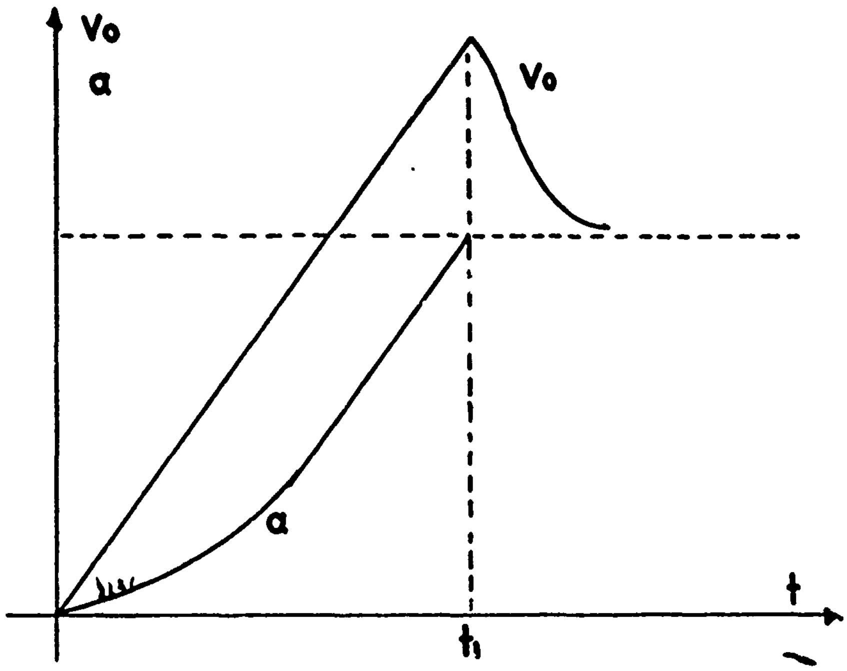


Figure 47. Input-output Plot of the L.P. Feedback Filter.

exceeded this level and is forced to drop back because of the regulating action.

Another important point is the difference in regulation observed between Figures 50 and 51 which should be expected because of the difference in feedback gains G_2 and therefore in "loop gain."

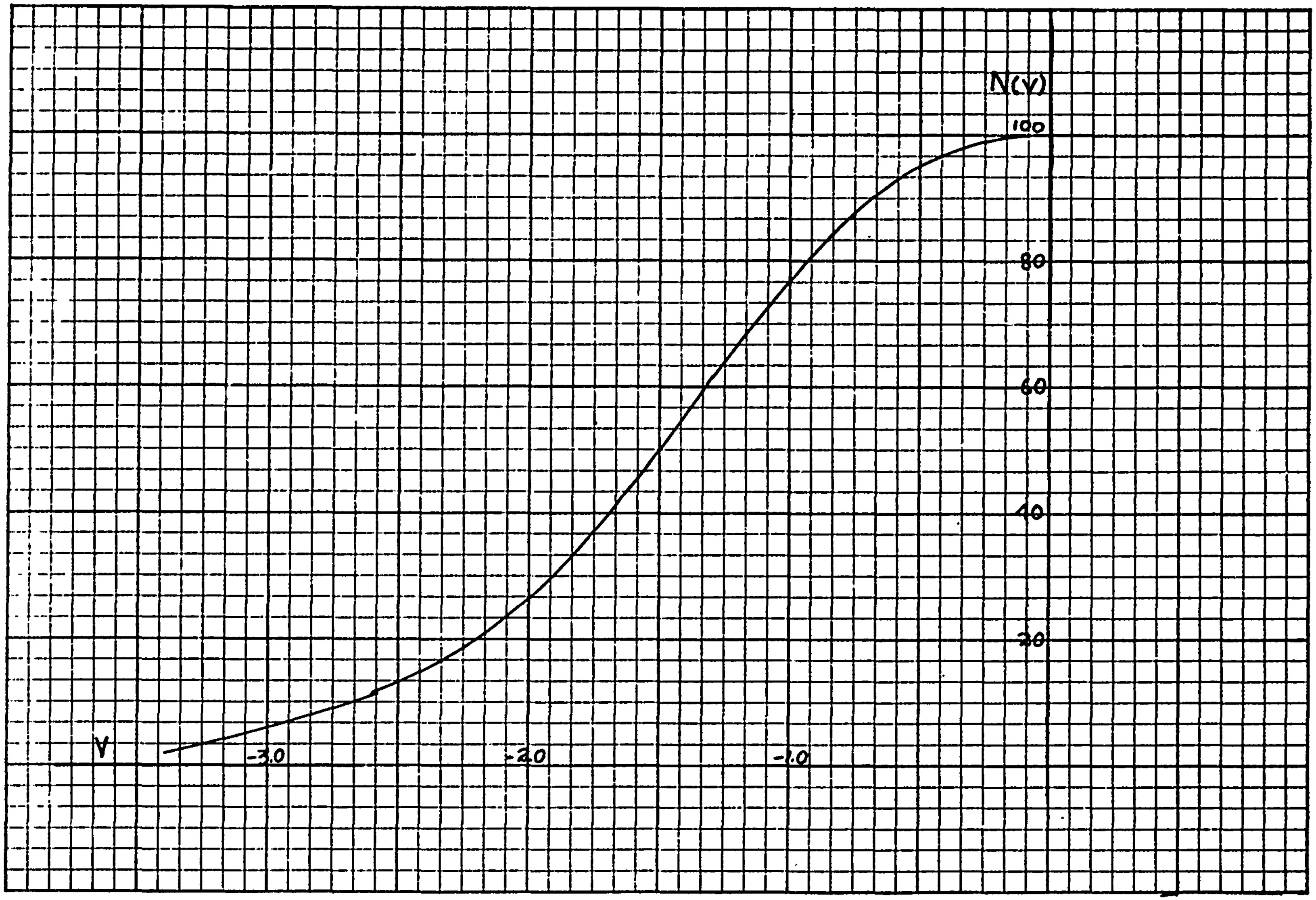
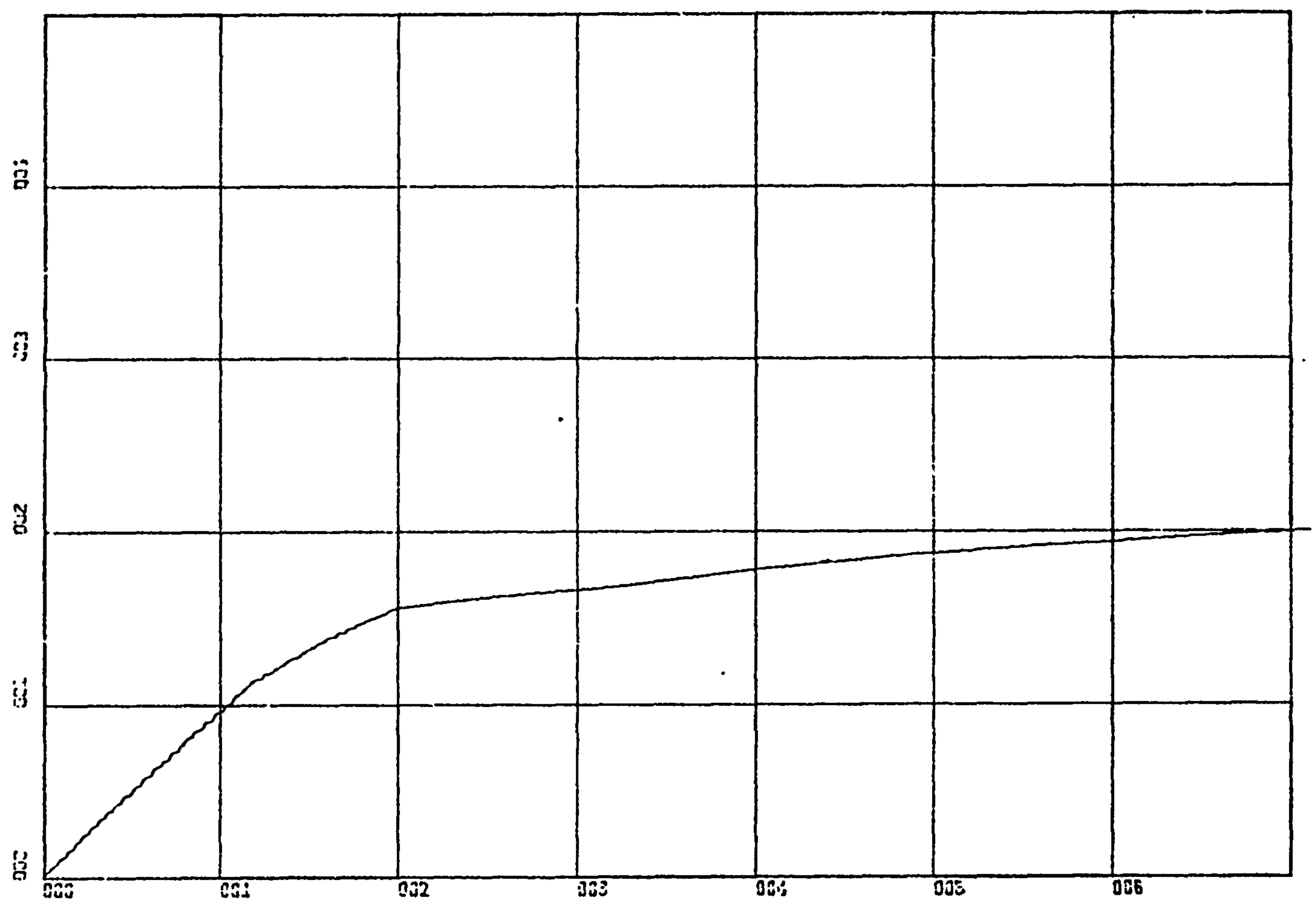
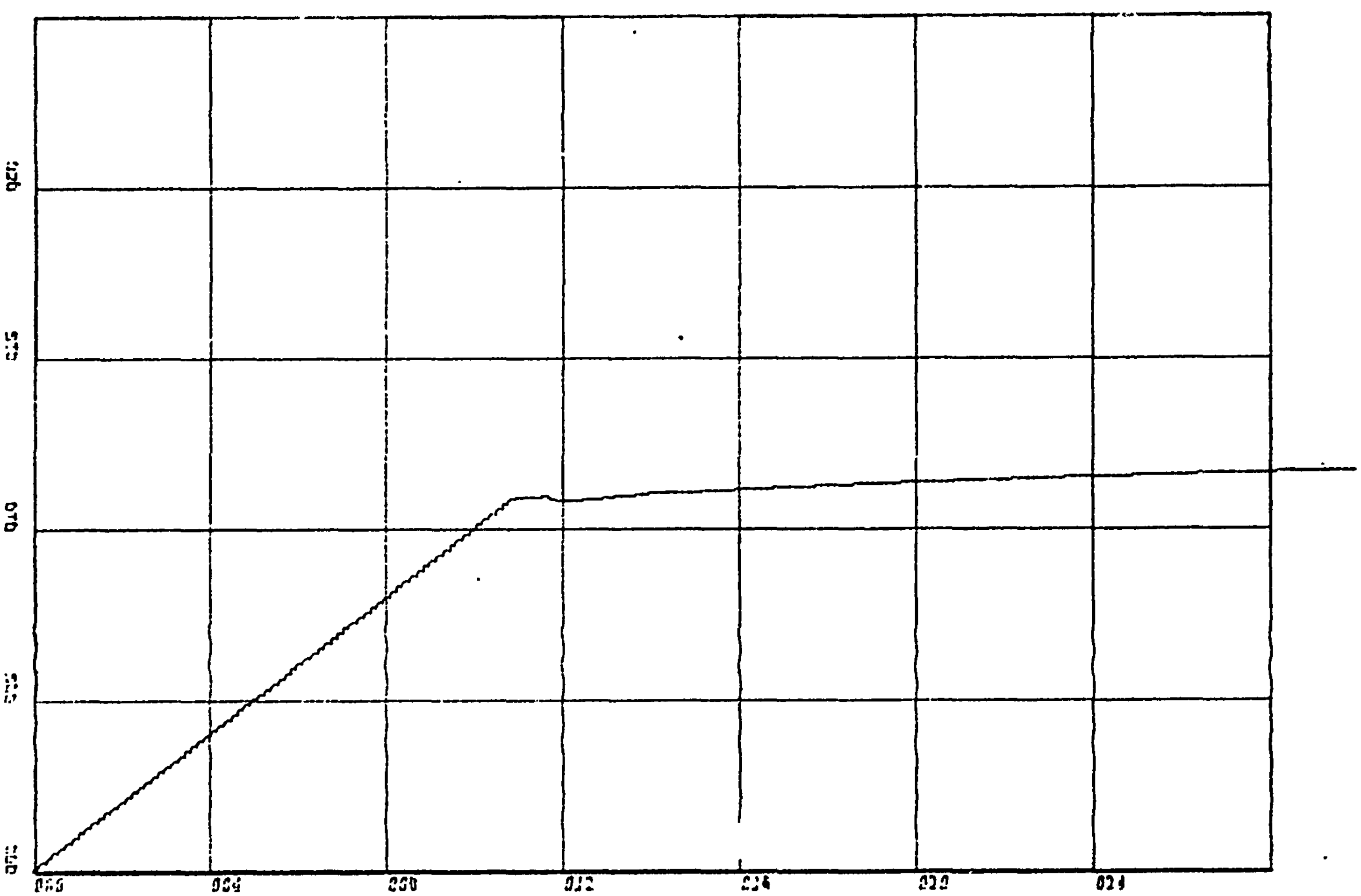


Figure 48. Gain Characteristic.



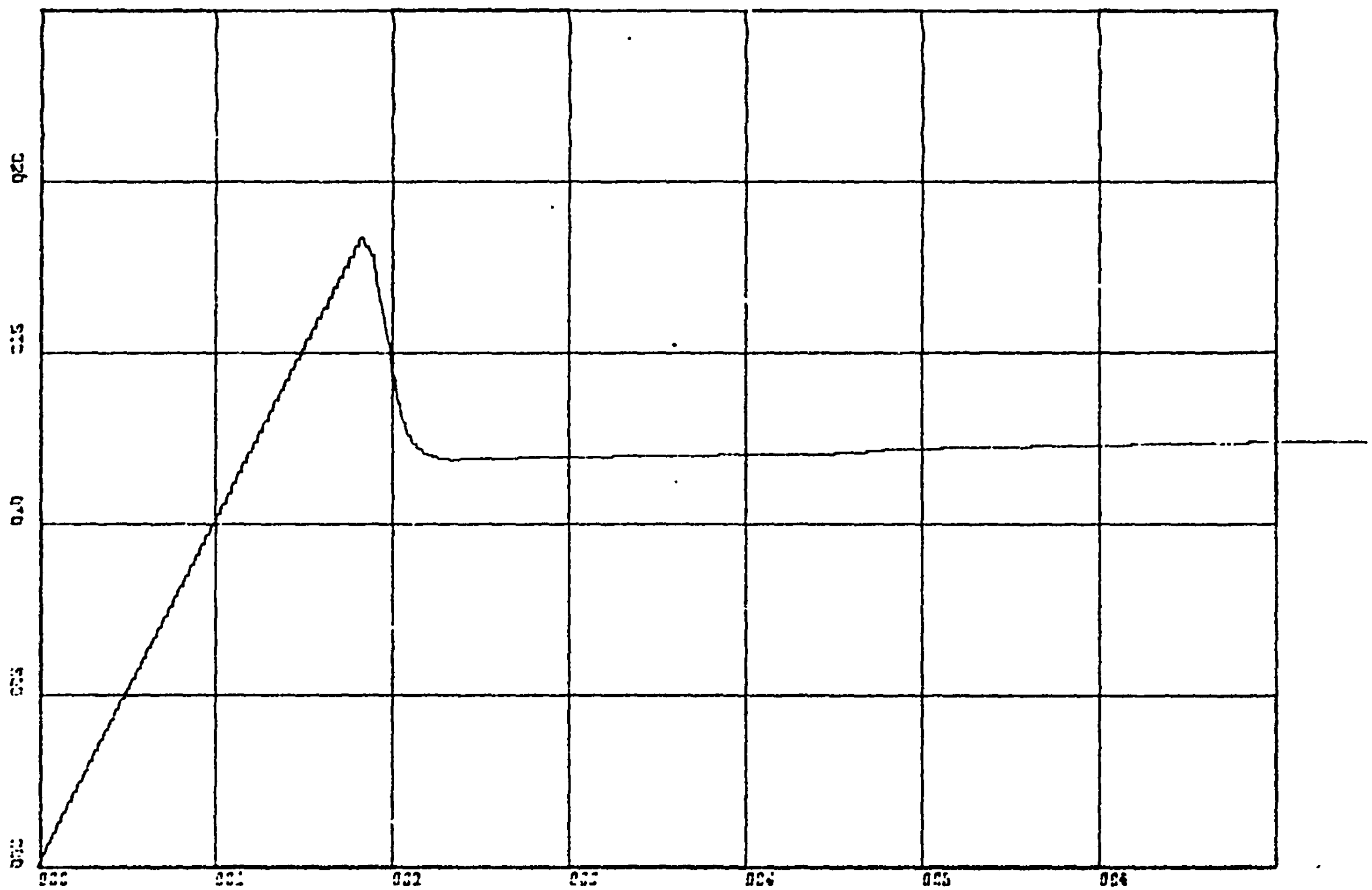
X: .01, Y: 1.0

Figure 49. Plot of Vo, G_2 = 1.0, V_{REF} = 0.0.



X: .04, Y: 5.0

Figure 50. Plot of Vo, G_2 = 1.0, V_{REF} = 10.0.



X: .01, Y: .5

Figure 51. Plot of Vo, G_2 = 10.0, V_{REF} = 10.0.

APPENDIX D

DERIVATION OF DISTORTION FORMULA

In Section III-F it was assumed that the amplifier transfer characteristic was given by:

$$T(v) = ao + a_1 v + a_2 v^2 + a_3 v^3 \qquad \text{(D-1)}$$

The gain is the slope of the transfer characteristic with respect to v and is

$$N = a_1 + 2a_2 v + 3a_3 v^2 \qquad \text{(D-2)}$$

The applied voltage to the grid of the amplifier is

$$v = v_1 + A[1+m \cos \omega_m t] \sin \omega_c t \qquad \text{(D-3)}$$

Then the gain is given substituting (D-3) into (D-2)

$$\begin{aligned} N &= a_1 + 2a_2 [v_1 + A(1+m \cos \omega_m t)\sin \omega_c t] + \\ &\quad + 3a_3 [v_1 + A(1+m \cos \omega_m t)\sin \omega_c t]^2 \\ &= a_1 + 2a_2 v_1 + 3a_3 v_1^2 + (2a_2 + 6a_3 v_1)A(1+\cos \omega_m t)\sin \omega_c t \\ &\quad + 3a_3 A^2 (1+m \cos \omega_m t)^2 \sin^2 \omega_c t \end{aligned} \qquad \text{(D-4)}$$

Then the output is given multiplying the input

$$Vin = A\,(1+m \cos \omega_m t)\sin \omega_c t$$

by the gain given in (D-4)

$$Vo = (a_1+2a_2v_1+3a_3v_1^2)\,A\,(1+m\cos\omega_m t)\sin\omega_c t$$

$$+ (2a_2+6a_3v_1)\,A^2\,(1+m\cos\omega_m t)^2\sin^2\omega_c t$$

$$+ 3a_3A^3\,(1+m\cos\omega_m t)^2\sin^3\omega_c t \qquad \text{(D-5)}$$

The second term of equation (D-5) contains the factor

$$\sin^2\omega_c t = \frac{1-\cos\omega_c t}{2}$$

so the final results of the expansion will be in the form

$$K_1+K_2\cos\omega_m t + K_3\cos 2\omega_m t + K_4\cos 2\omega_c t + K_5\cos\omega_m t\ \cos 2\omega_c t$$

$$+ K_6\cos 2\omega_m t\ \cos 2\omega_c t$$

which are all terms not contributing to the desired output waveform and therefore are supposed to filtered out.

In the third term of equation (D-5) the factor $\sin^3\omega_c t$ may be replaced as follows:

$$\sin^3\omega_c t = \frac{3\sin\omega_c t - \sin 3\omega_c t}{4}$$

and the third term becomes

$$\frac{3a_3A^3}{4}\,[3\sin\omega_c t - \sin 3\omega_c t]\,[1+3m\cos\omega_m t + 3m^2\cos^2\omega_m t$$

$$+ m^3\cos 3\omega_m t] = \frac{3a_3A^3}{4}\,[3\sin\omega_c t - \sin 3\omega_c t]\,\{1+3m\cos\omega_m t$$

$$+ \frac{3m^2}{2}(\cos^2\omega_m t+1) + \frac{m^3}{4}(\cos 3\omega_m t + 3\cos\omega_m t)\} \qquad \text{(D-6)}$$

Now filtering out of equation (D-6) all the terms which do not contribute to the desired wave form the following remains:

$$\frac{3a_3A^3}{4}\cdot 3\sin\omega_c t\left[1+3m\cos\omega_m t+\frac{3m^2}{2}+\frac{3m^3}{4}\cos\omega_m t\right]$$

$$= A\left[\frac{9a_3A^2}{4}\left(1+\frac{3m^2}{2}\right)+\frac{27a_3A^2m}{4}\left(1+\frac{m^2}{4}\right)\cos\omega_m t\right]\sin\omega_c t \tag{D-7}$$

Adding equation (D-7) with the first term of equation (D-5) it is obtained:

$$Vo = A\left[a_1+2a_2v_1+3a_3v_1^2+m(a_1+2a_2v_1+3a_3v_1^2)\cos\omega_m t + \frac{9a_3A^2}{4}\left(1+\frac{3m^2}{2}\right)+\frac{27a_3A_m^2}{4}\left(1+\frac{m^2}{4}\right)\cos\omega_m t\right]\sin\omega_c t$$

$$= A\sin\omega_c t\left\{a_1+2a_2v_1+3a_3v_1^2+\frac{9a_3A^2}{4}\left(1+\frac{3m^2}{2}\right) + m\left[a_1+2a_2v_1+3a_3v_1^2+\frac{27a_3A^2}{4}\left(1+\frac{m^2}{4}\right)\right]\cos^2\omega_m t\right\}$$

$$= \frac{A}{a_1+2a_2v_1+3a_3v_1^2+\frac{9a_3A^2}{4}\left(1+\frac{3m^2}{2}\right)}\left\{1 + m\left[\frac{a_1+2a_2v_1+3a_3v_1^2+\frac{27a_3A^2}{4}\left(1+\frac{m^2}{4}\right)}{a_1+2a_2v_1+3a_3v_1^2+\frac{9a_3A^2}{4}\left(1+\frac{3m^2}{2}\right)}\right]\cos\omega_m t\right\}\sin\omega_c t$$

$$= B\left[1+m'\cos\omega_m t\right]\sin\omega_c t \tag{D-8}$$

which is the desired form of the output wave form. Hence

$$\frac{m'}{m} = \frac{a_1+2a_2v_1+3a_3v_1^2+\frac{27}{4}a_3A^2\left(1+\frac{m^2}{4}\right)}{a_1+2a_2v_1+3a_3v_1^2+\frac{9}{4}a_3A^2\left(1+\frac{3}{2}m^2\right)}$$

$$= 1+\frac{\frac{9}{4}a_3A^2\left(1-\frac{3}{4}m^2\right)}{a_1+2a_2v_13a_3v_1^2+\frac{9}{4}a_3A^2\left(1+\frac{3}{2}m^2\right)}$$

which is identical to equation (3-82).

APPENDIX E

EXAMPLE 1 - COMPENSATION

In this example the system of Appendix B is considered. As it was derived by the theory and proved by simulation there, the system was unstable for input amplitudes exceeding $\overline{Vin}$ = .11. A compensation will be attempted here to increase the range of input amplitudes for which the system remains stable.

A "lag" filter is going to be used in the forward path with pole at ω = 2 rad and zero at ω = 20 rad. The transfer function of the filter is $\frac{\frac{1}{20}s+1}{\frac{1}{2}s+1}$ and as it is well known the loop gain is not affected by this insertion.

The new block diagram of the system is as shown in Fig. 52 and the BODE diagram of the compensated system is shown in Fig. 53 where only the magnitude asymptotes have been drawn.

From the BODE diagram which is drawn for an input amplitude to $\overline{Vin}$ = 0.5 after the insertion of the lag filter it seems the gain margin is raised to about 26 db or a numerical gain of 20.

This means that the input amplitude can be raised up to Vin = .05 x 20 = 1. before the system reaches again a limit cycle.

Simulations of the compensated system were tried for input amplitudes .4, .6, .8, 1.2, and the resulting

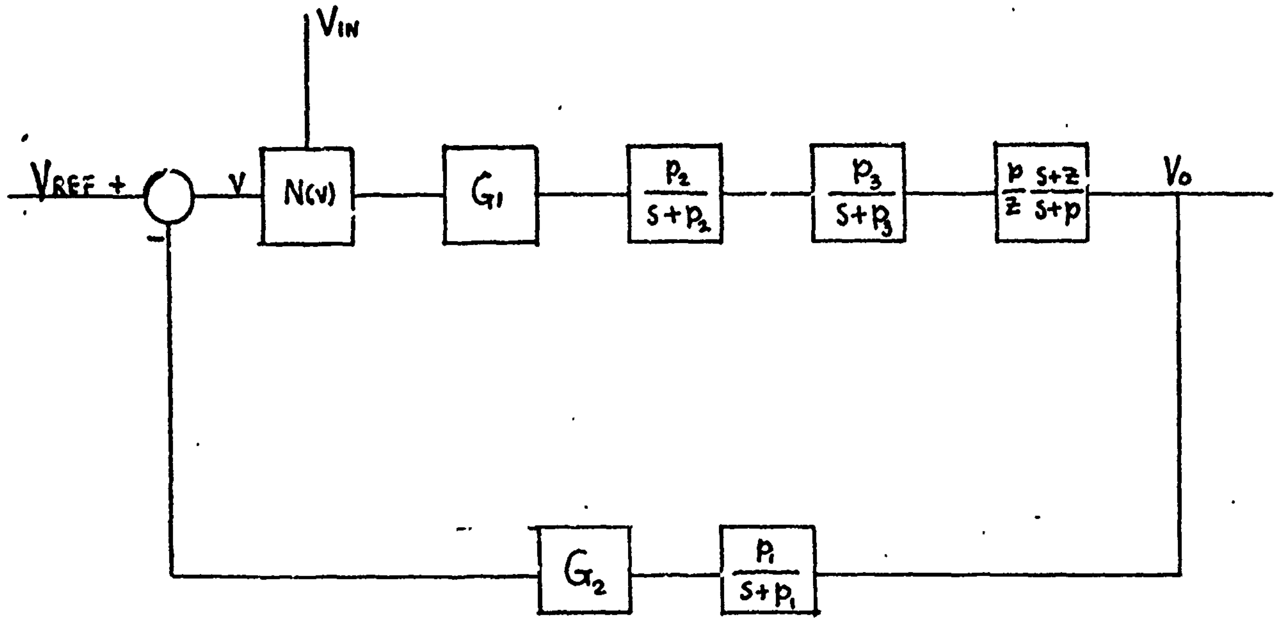


Figure 52. Block Diagram of the Compensated System.

outputs vs time are shown in the following figures .

Input amplitudes closer to the critical value $\overline{Vin}$ = 1.0 were avoided because although every input amplitude less than 1.0 yields a stable system, the response becomes very oscillatory and the computer runs should be very long in order for the system to stabilize.

For input amplitudes .4 and .8 two simulations were tried having a bias voltage in the feedback path in a sense of initial condition.

The comparison of the corresponding output plots shows how the settling time and peak overshoot have been reduced by this technique. Note that the abscissa scale of the last two simulations is half of the previous ones.

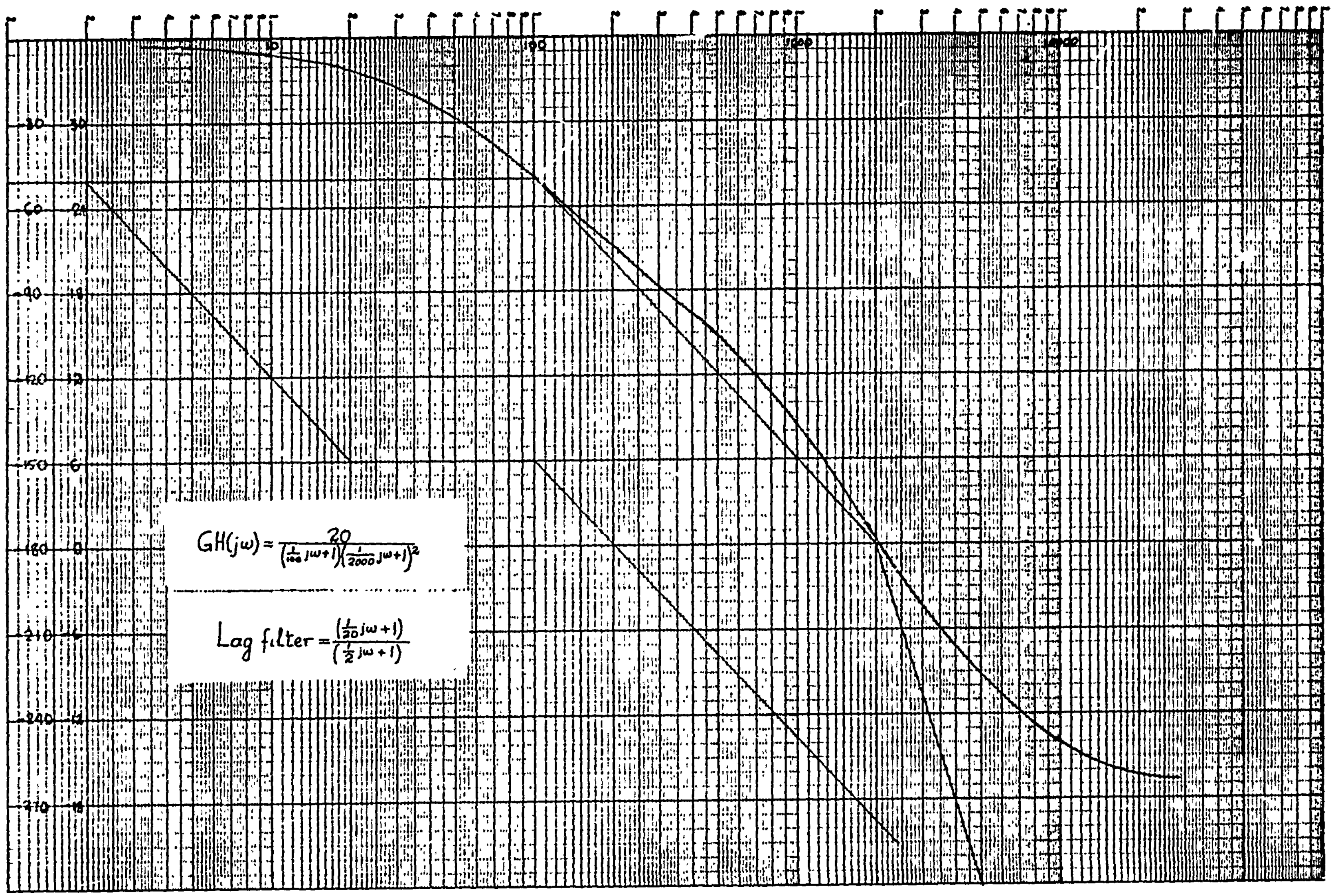
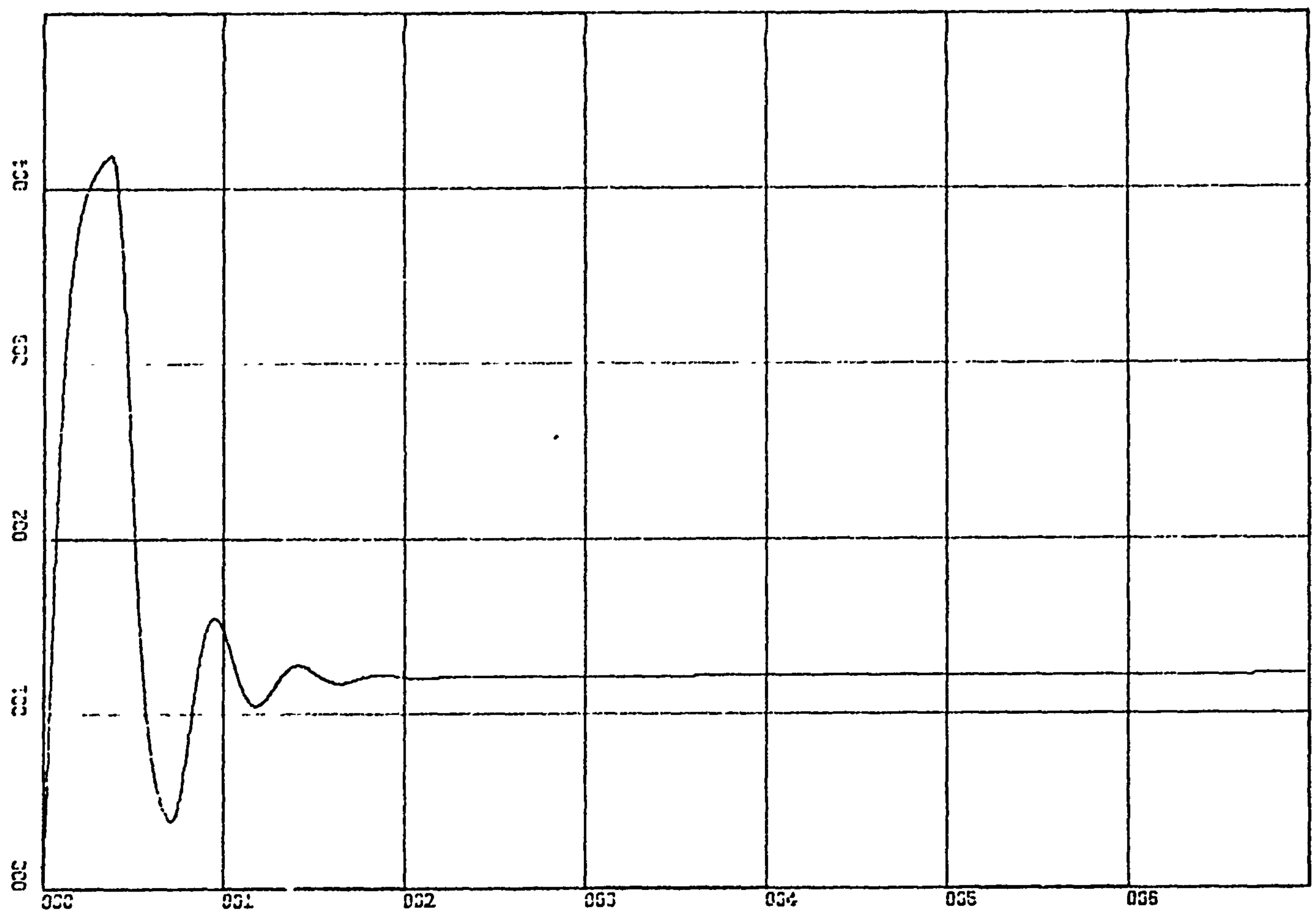
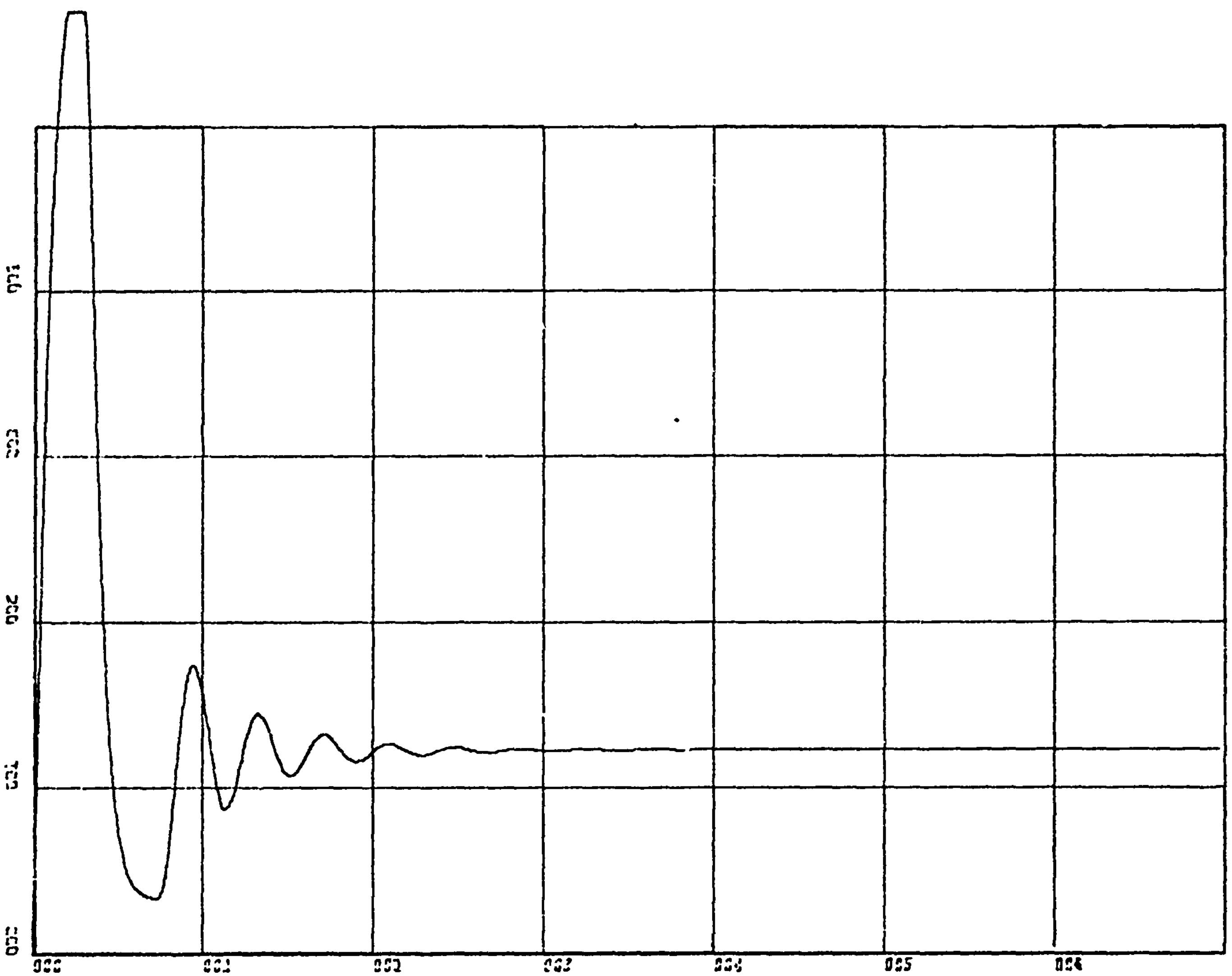


Figure 53. BODE Plot of Compensated System.



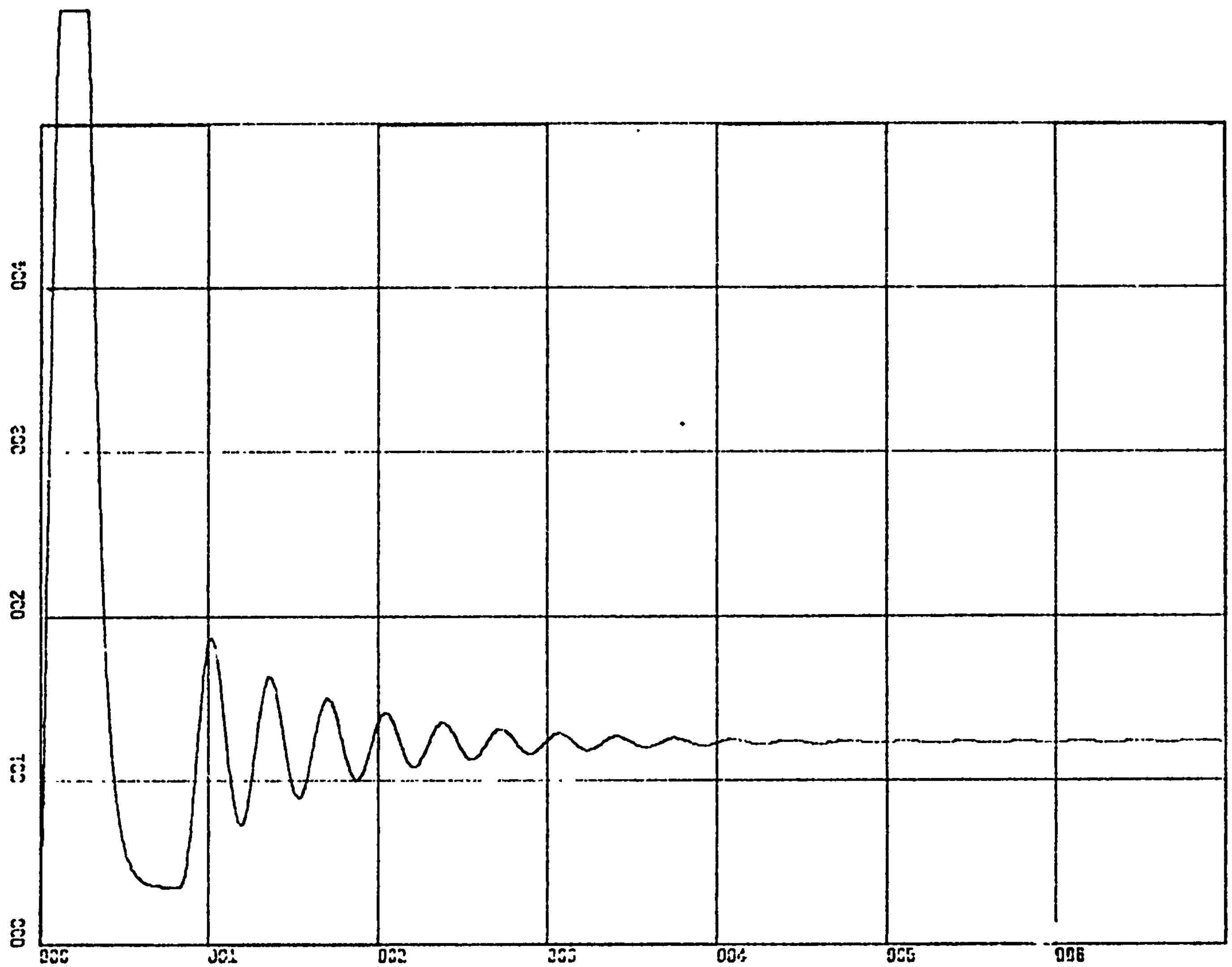
X: .01, Y: 1.0

Figure 54. Plot of Vo, Vin = .4.



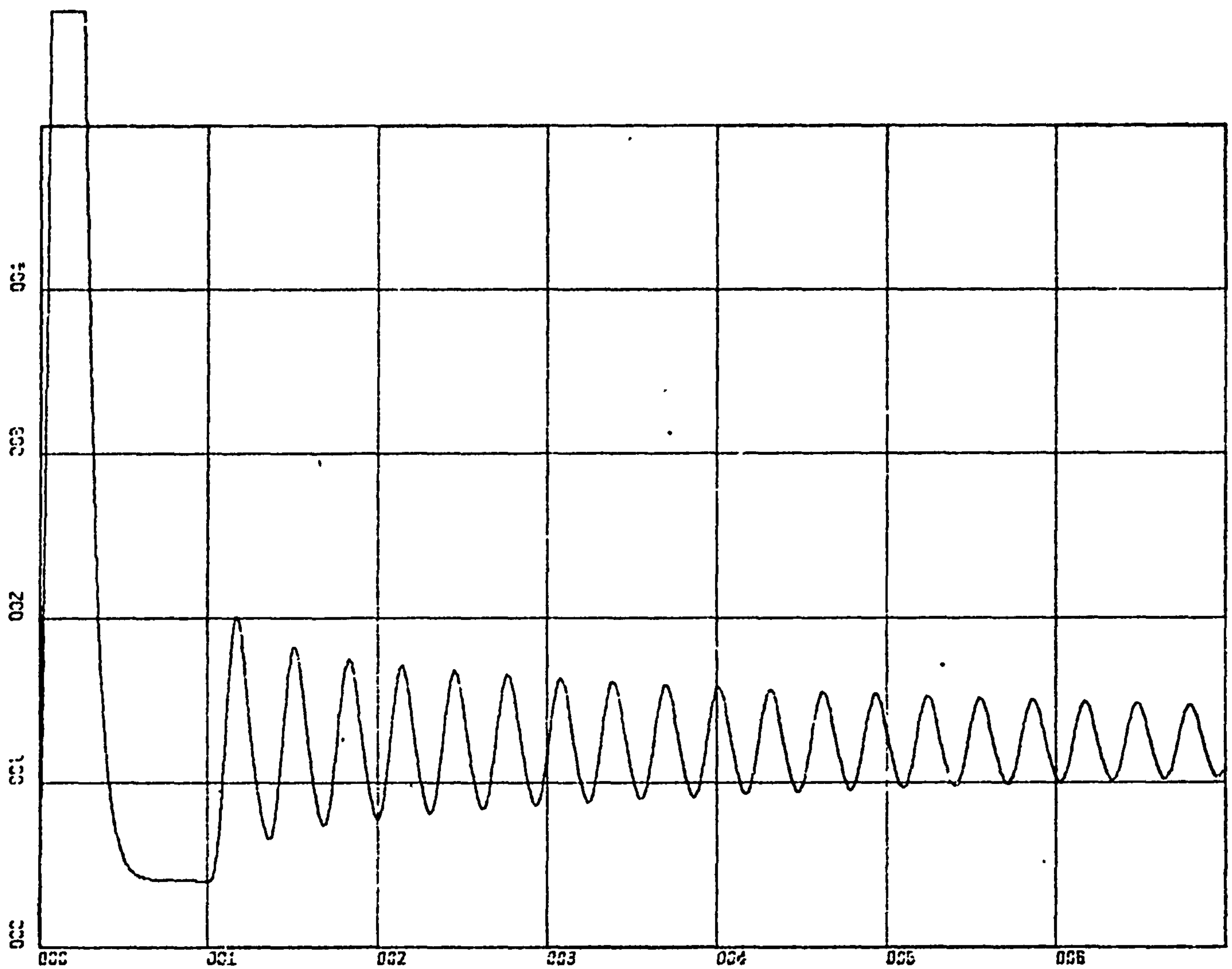
X: .01, Y: 1.0

Figure 55. Plot of Vo, Vin = .6.



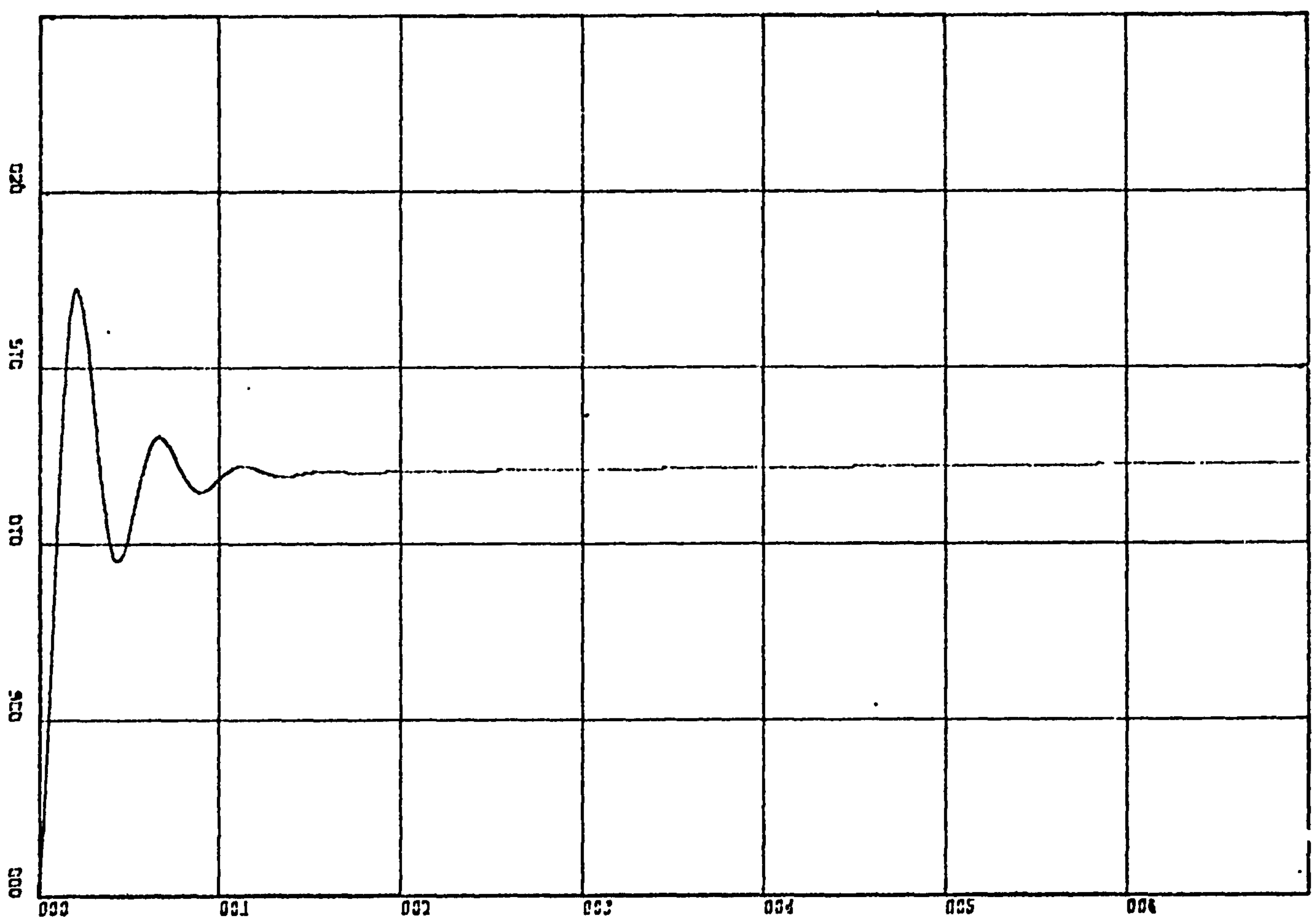
X: .01, Y: 1.0

Figure 56. Plot of Vo, Vin = .8.



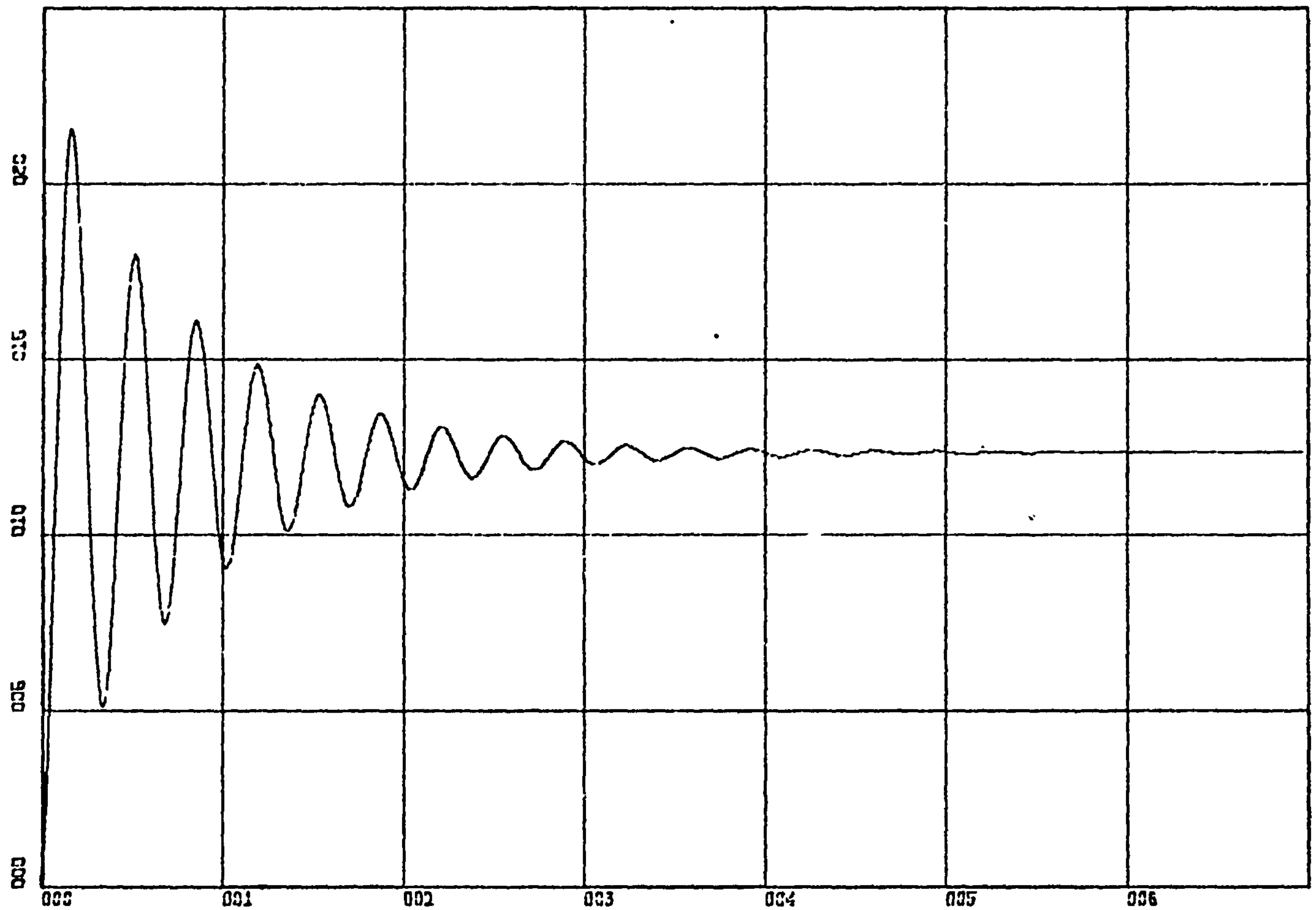
X: .01, Y: 1.0

Figure 57. Plot of Vo, Vin = 1.2.



X: .01, Y: .5

Figure 58. Plot of Vo, Feedback Filter with Bias, Vin = 0.4.



X: .01, Y: .5

Figure 59. Plot of Vo, Feedback Filter with Bias, Vin = 0.8.

EXAMPLE 2 - RESHAPING OF TRANSIENT RESPONSE

Suppose the system whose block diagram is shown is in Fig. 60 is given, with non-linear gain N(v) versus bias voltage characteristic as shown in Fig. 61. The question is imposed as to what range of input amplitude can be regulated into 1 db range ($\bar{V}o \pm 1/2$ db) of output amplitude.

It is observed that the coefficients have been chosen in such a way as $\lim_{s \to 0} GH(s) = 1$ which is always the assumption for the loops considered so far. If the actual system does not comply with the above assumption it can always be brought into that form.

The first thing to be examined is for what loop gain the system becomes unstable. This can be done by any conventional linear feedback technique. Root locus will be used in this case.

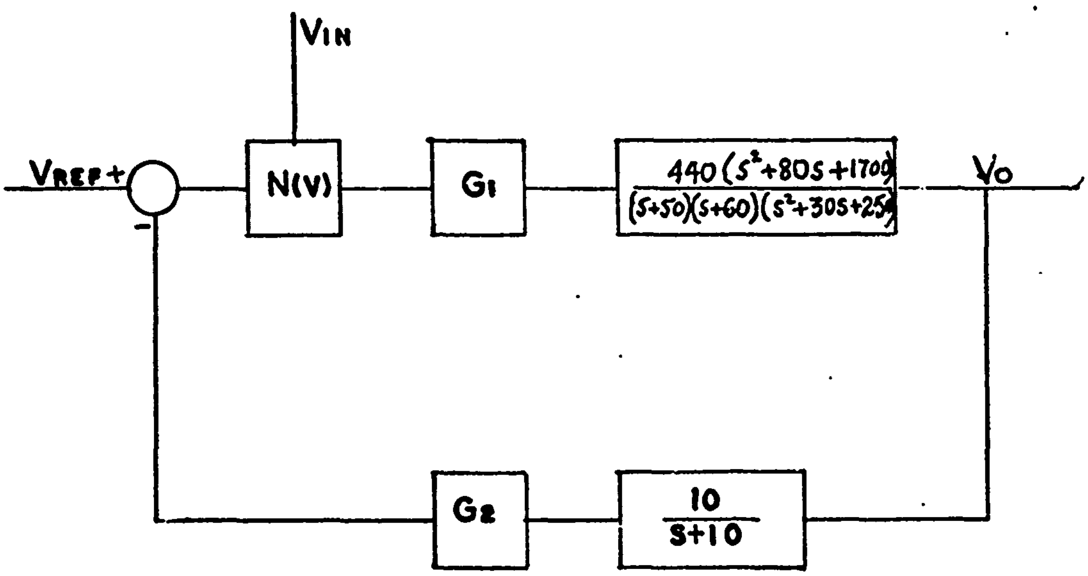


Figure 60. Block Diagram of AGC System for Example 2.

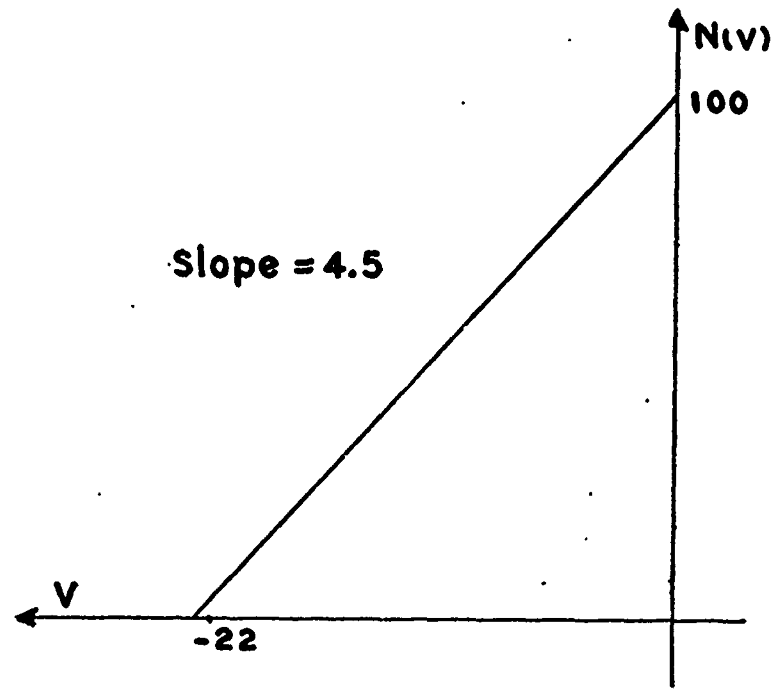


Figure 61. Non-linear Gain Characteristic for Example 2.

The characteristic equation of the system is

$$1 + KGH(s) = 0$$

or

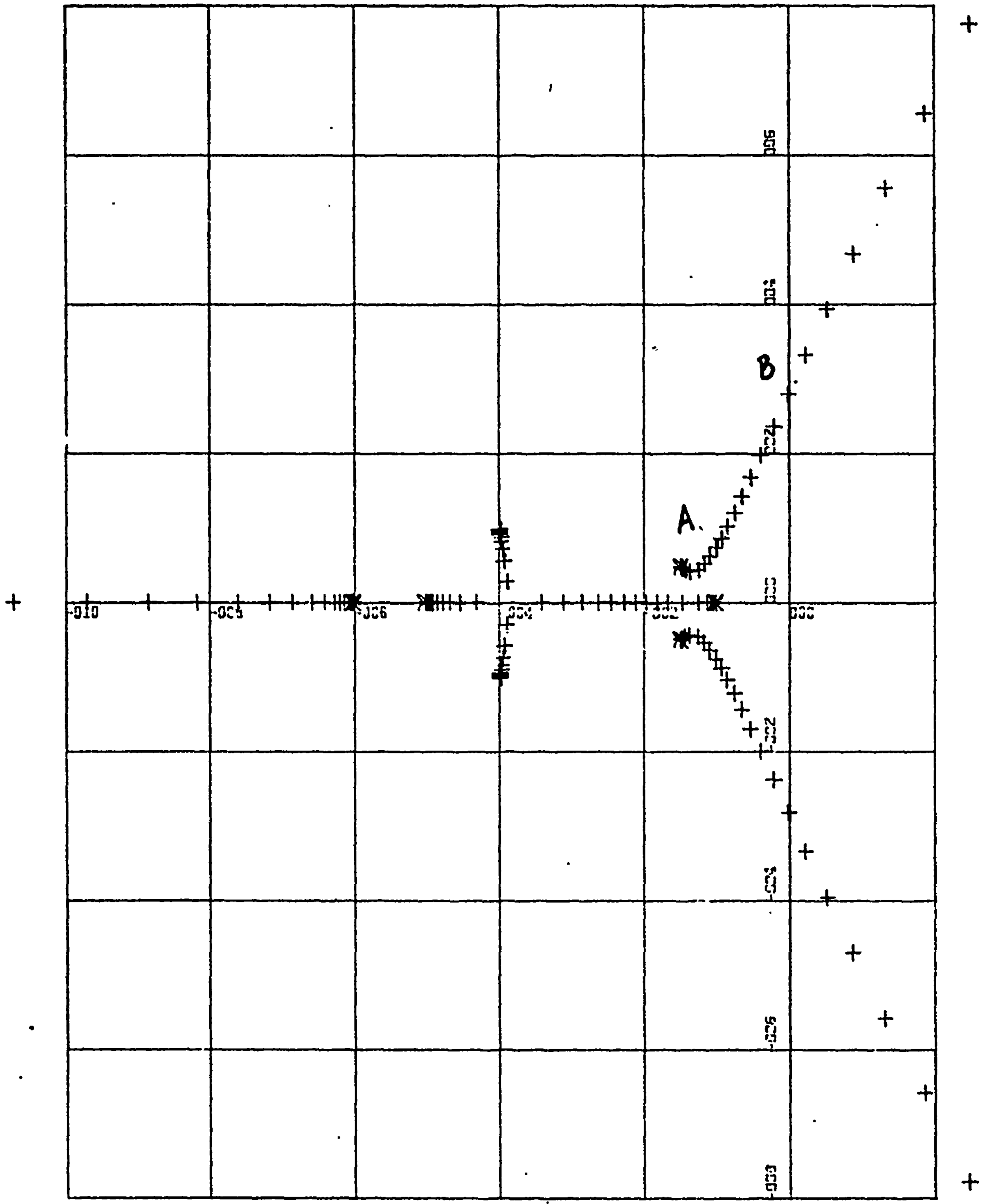
$$1 + \frac{K(s^2+80s+1700)}{(s+10)(s+50)(s+60)(s^2+30+250)} = 0$$

or

$$s^5 + 150s^4 + 7950s^3 + (183000+K)s^2 + (1925000+80K)s + (7500000 + 1700K) = 0.$$

The root locus for this characteristic equation is shown plotted in Fig. 62.

Comparing the plot and the computer print-out it is found that for a value of gain $K = 4.64 \times 10^4$ the roots enter the right hand S-plane and the system becomes unstable.



X: 20.0, Y: 20.0

Figure 62. Root Locus of Uncompensated System.

Since the system contains already a factor of gain 4.4×10^3, it is seen that there exists a gain margin of $\frac{4.64 \times 10^4}{4.4 \times 10^3} = 10.5$ which can be used as loop gain by the AGC system before the loop becomes unstable.

Substituting numbers into the formula (3-15)

$$10.5 = \frac{0.115\ \overline{V}o\ [\text{Gain change in db}]}{\overline{V}o(1.06-.94)}$$

and solving

$$[\text{Gain change in db}] \approx 10.5$$

Therefore it seems the system would be able to regulate 11.5 db input amplitude range into 1 db output amplitude range, (Gain change = 10.5 db) and in order not to consider the case of having a root exactly on the $j\omega$ axis, one can consider 10 db input range into 1 db output range.

As can be seen on the root locus plot, the behavior of the system is primarily governed by the pair of roots closer to the origin, the other roots being away from the origin and having little or no effect on the transient response of the system. The operating region on which this pair of roots will move during the operation as the input amplitude will change is indicated on the root locus as AB.

Another characteristic which can be derived, is that the operation takes place very close to the $j\omega$ axis having as a result very oscillatory transient response and excessively long settling time. Because of this it is inferred that a continuously changing input amplitude has to be of

very low frequency in order the system to be able to follow and regulate it, otherwise for higher frequency the loop will operate continuously in a transient condition without being able to get to steady-state and naturally the regulation will not take place.

In order to demonstrate the above thoughts suppose the input is to vary from 170 mV to 530 mV (10 db range) and $G_1 = 1.0$. Suppose also the output is desired to be maintained between 4.7 and 5.3 volts (5 volts $\pm$ 1/2 db). If other values of output amplitude are desired, a linear amplifier or attenuator can be placed after the loop, or the linear gain G_1 can be used.

Since the input and output amplitudes have been specified, the operating region of the non-linear gain characteristic can be readily calculated.

$$\text{Nmin} = \frac{\text{Vomax}}{\text{Vinmax}} = \frac{5.3}{.530} = 10$$

$$\text{Nmax} = \frac{\text{Vomin}}{\text{Vinmax}} = \frac{4.7}{.170} = 28.2$$

corresponding to vmax = -20, vmin = -15.9 respectively (making use of $N(v) = 100 + 4.5v$). Then the feedback linear gain is calculated

$$G_2 = \frac{\Delta v}{\Delta Vo} = \frac{4.1}{.6} = 6.85$$

and

$$V_{REF} = G_2\text{Vomin} + \text{vmin} = 6.85 \times 4.7 - 15.9 = 16.2$$

(Making use of equations (3-67) and (3-69) correspondingly.)

The so formed AGC system is going to be tested applying first two step-inputs with amplitudes 250 mV and 450 mV. The corresponding computer outputs of the simulation are shown in Figures 63 and 64.

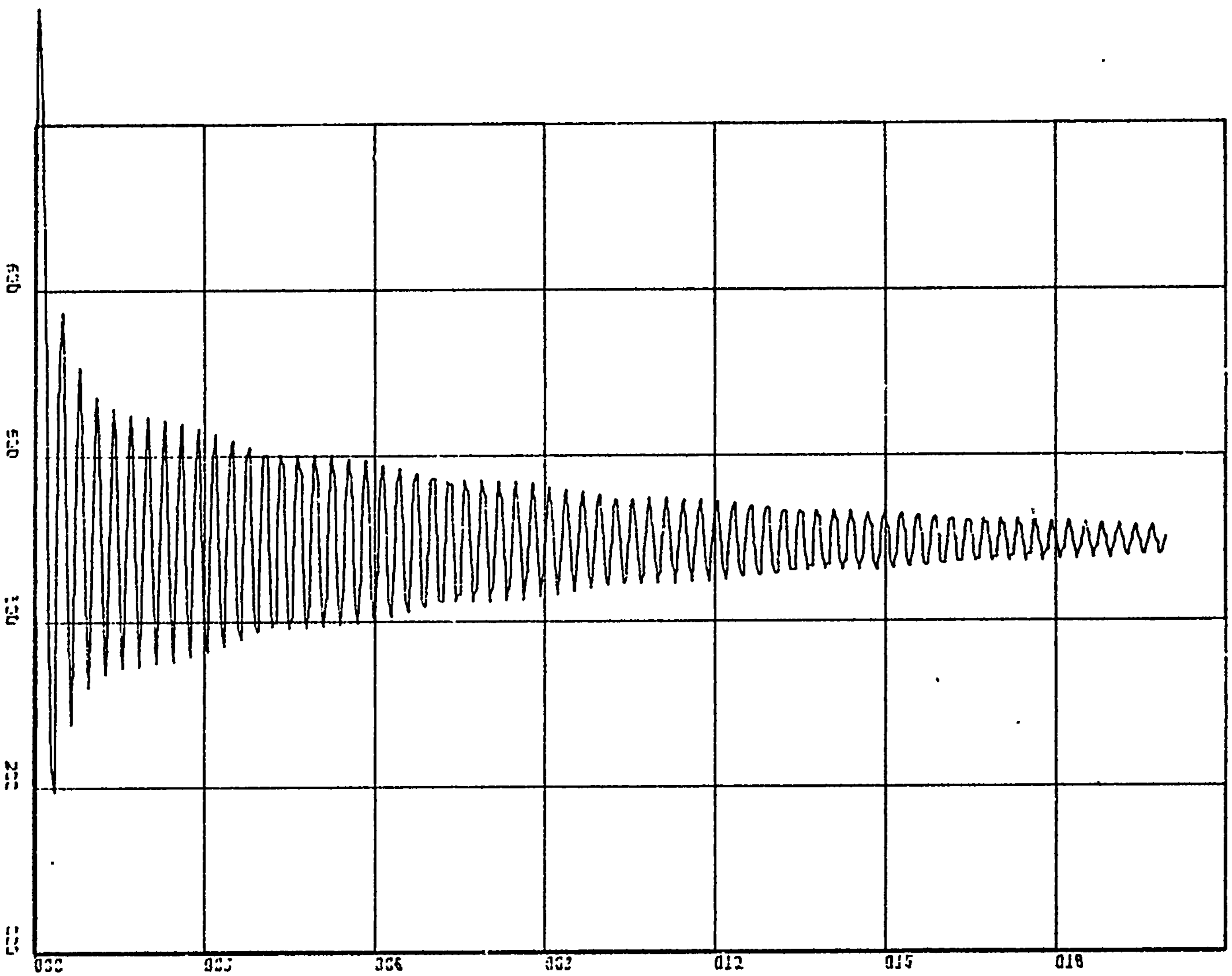
It is there seen that although a computer output of 20 seconds was asked the AGC output has not yet gotten into the steady-state, continuing oscillating around a mean value in the interval of 4.7 to 5.3 of output amplitude range.

As a second test a sine wave of amplitude 180 mV oscillating about a mean value of 350 mV with a frequency of 1.0 rad/sec is going to be applied. The frequency is well outside the frequency band of the closed loop so it should be attenuated, and the amplitude oscillate between 170 mV and 530 mV the assumed input range.

The corresponding computer output for the simulation is seen in Fig. 65. As it is observed there the system never gets to steady-state and the output consists of a continuous transient wave modulated by the input sine wave.

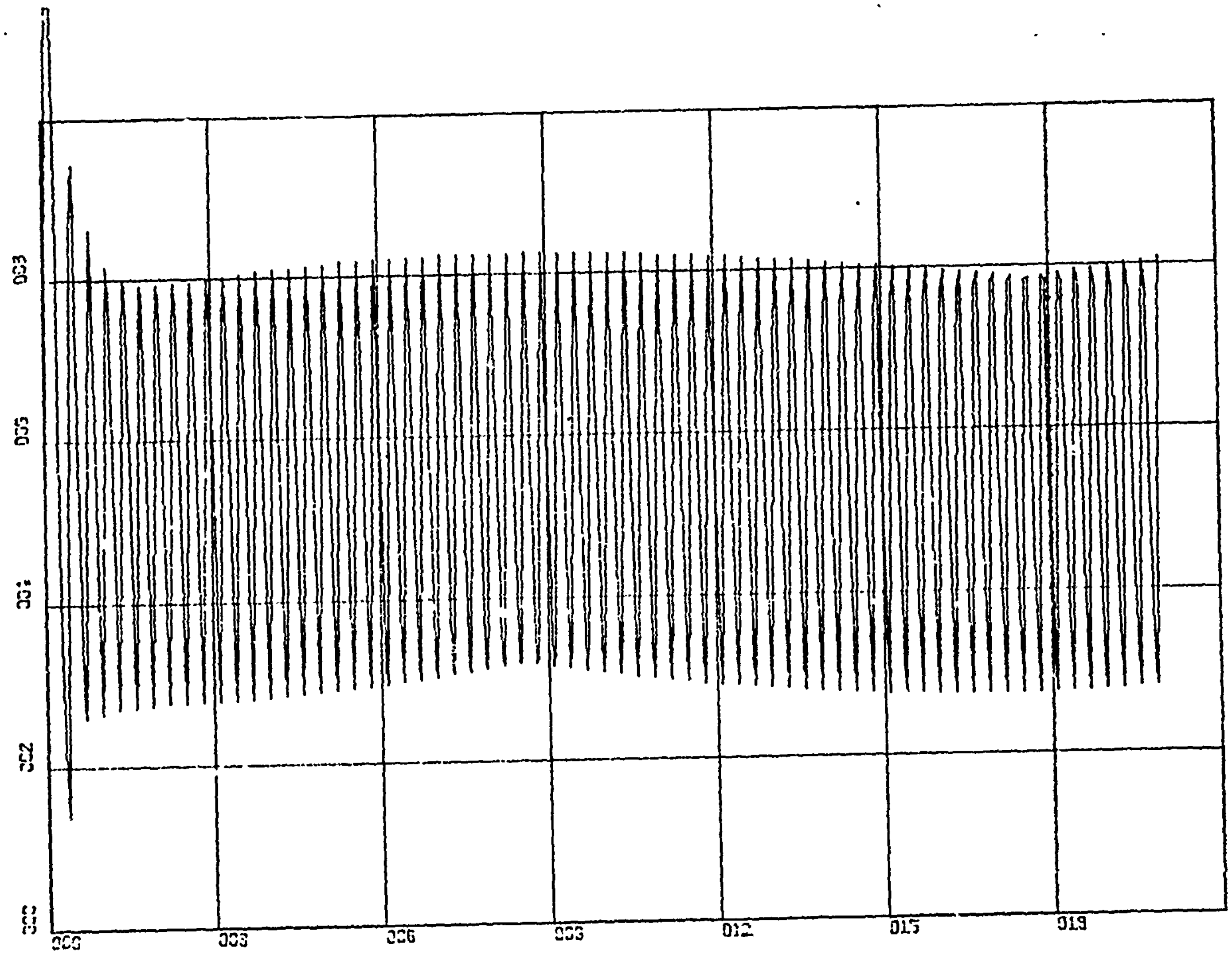
The regulation is poor exceeding the predetermined limits of the output amplitude.

As a means of reshaping the transient response and making the settling time faster a lead filter in the forward path is going to be used, with distance between zero and pole one decade. In order to keep the loop gain constant in the presence of the filter attenuation an additional gain of 10 has been added.



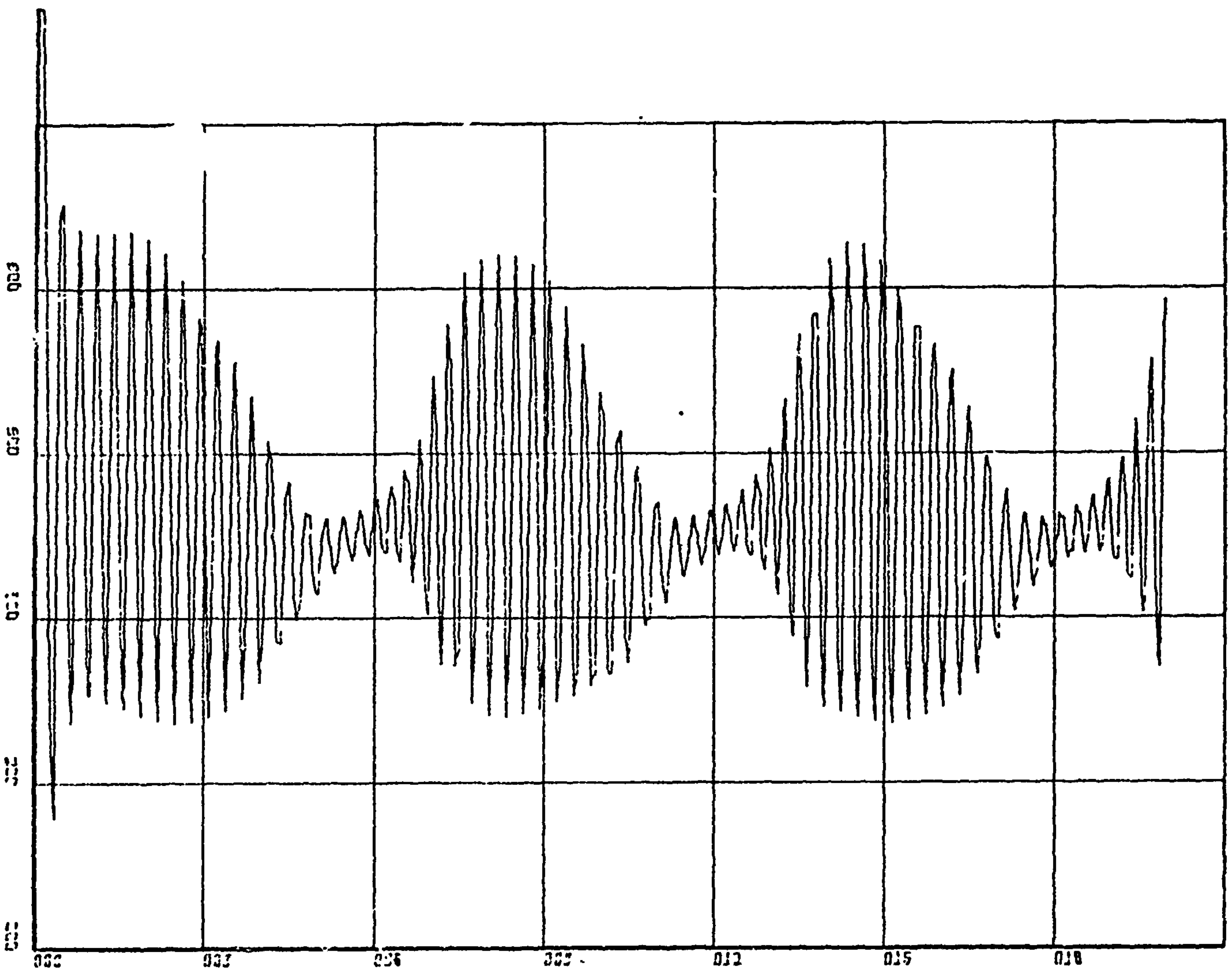
X: 3.0, Y: 2.0

Figure 63. Plot of Vo, Uncompensated System, Vin = 0.25.



X: 3.0, Y: 2.0

Figure 64. Plot of Vo, Uncompensated System, Vin = 0.45.



X: 3.0, Y: 2.0

Figure 65. Plot of Vo, Uncompensated System, Input Waveform Sinusoidal.

The transfer function of the lead filter was $\frac{10(s+10)}{s+100}$.
The characteristic equation became

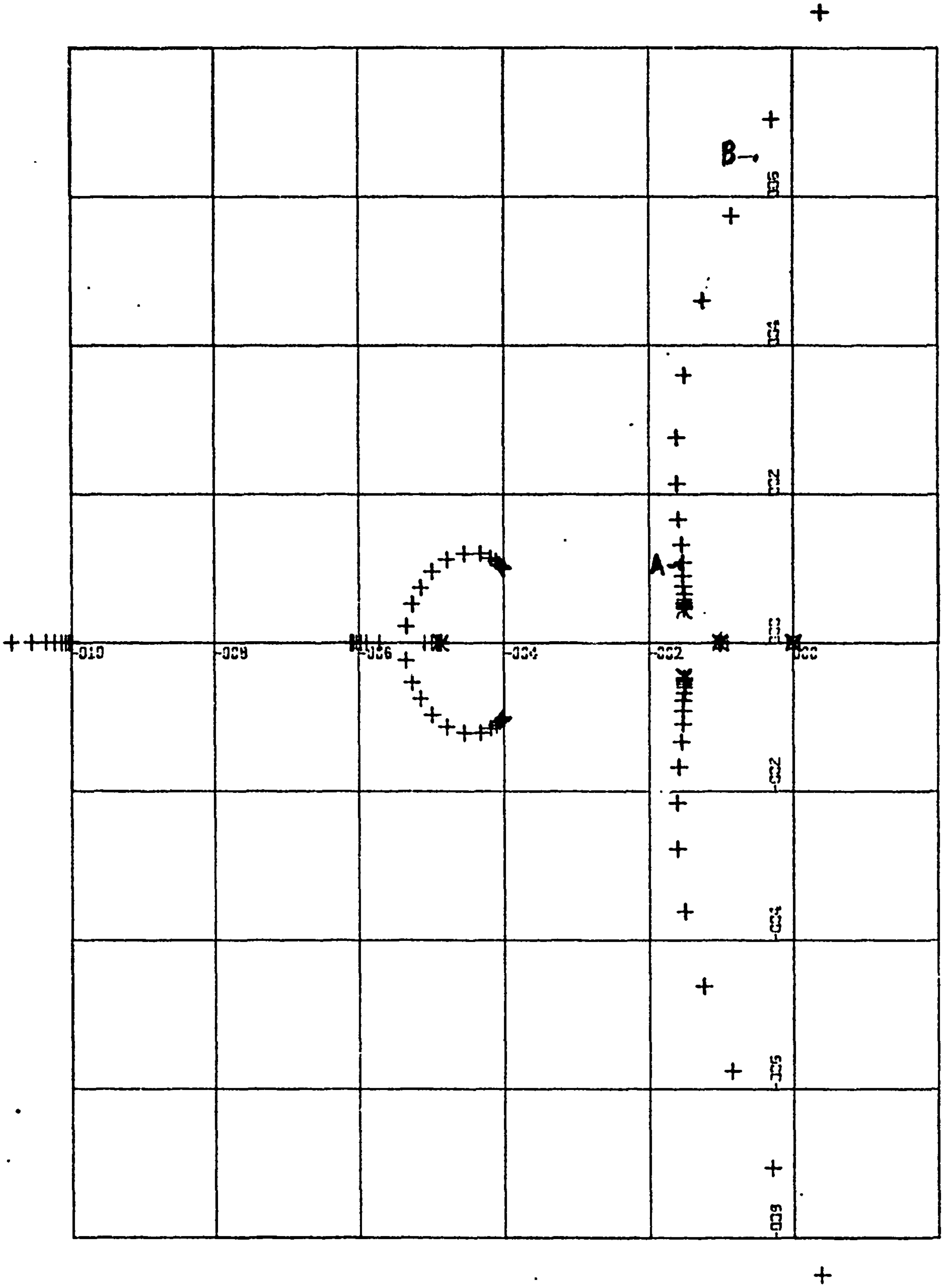
$$1+GH(s) = 1 + \frac{10K(s^2+80s+1700)(s+10)}{(s+10)(s+50)(s+60)(s^2+30s+250)(s+100)} = 0$$

or

$$s^6 + 2.5 \times 10^2 s^5 + 2.295 \times 10^4 s^4 + (9.78 \times 10^5 + 10K)s^3$$
$$+ (2.024 \times 10^7 + 9.0 \times 10^2 K)s^2 + (2 \times 10^8 + 2.5 \times 10^4 K)s$$
$$+ (7.5 \times 10^8 + 1.7 \times 10^5 K) = 0$$

The root locus for this characteristic equation is shown in Fig. 66 and the gain at which the system becomes unstable turns out to be $K = 8.989 \times 10^4$. The operating region on the root locus is indicated as AB and it is seen that the existing gain margin before the system gets unstable has increased by a factor of almost 2. The pair of dominant poles is now sufficiently far from the $j\omega$ axis predicting a satisfactory transient response and a much faster settling time. Quantitative results cannot be predicted even for one particular input amplitude, as it could be thought, because although the operation point can be found, the system is highly non-linear, the gain N(v) changing with the transient of the output and it is well known that results are available for only linear systems.

Qualitative, though, predictions can be made by the place of the operating region on the s-plane as in the present case.



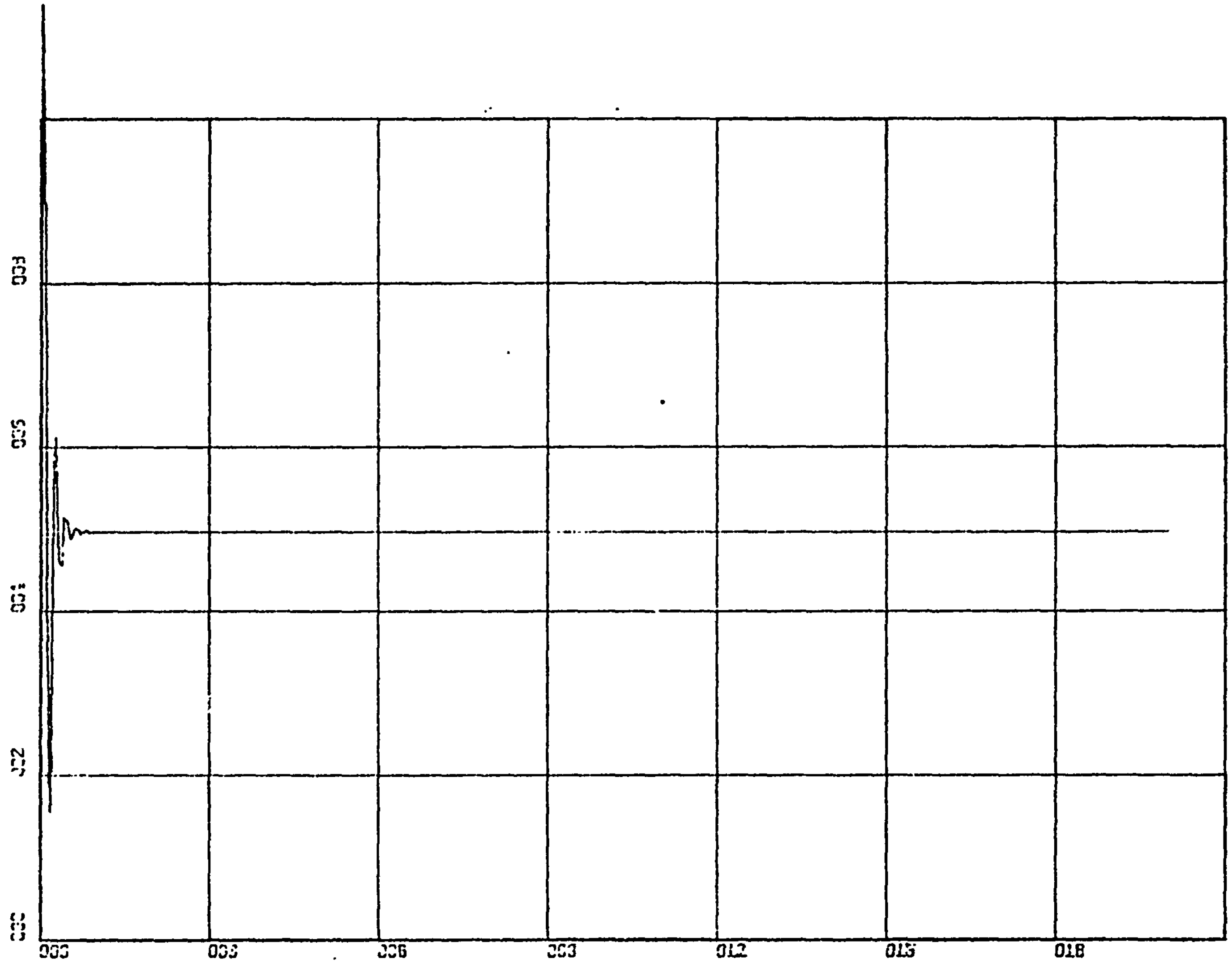
X: 20.0, Y: 20.0

Figure 66. Root Locus of Compensated System.

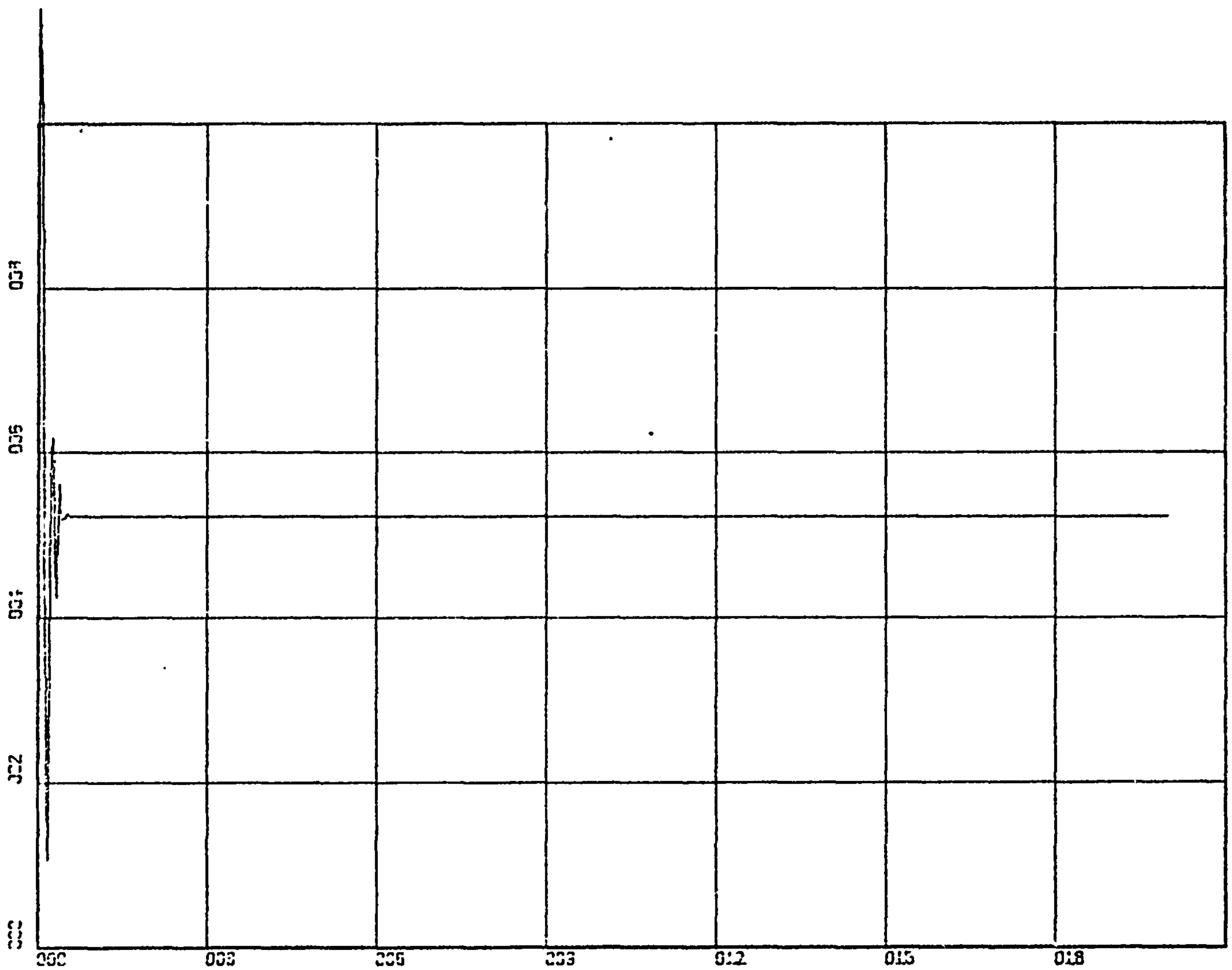
The compensated system was tested for the same inputs used in the uncompensated loop.

The simulation output the two steps in indicated in Fig. 67 and 68, showing a very good transient response.

As a result the output of the sine wave is shown in Fig. 69 where it is seen that after the first transient the output settles down and the regulation is excellent, the output amplitude taking values exactly in the predicted limits following the sinusoidal shape of the input.

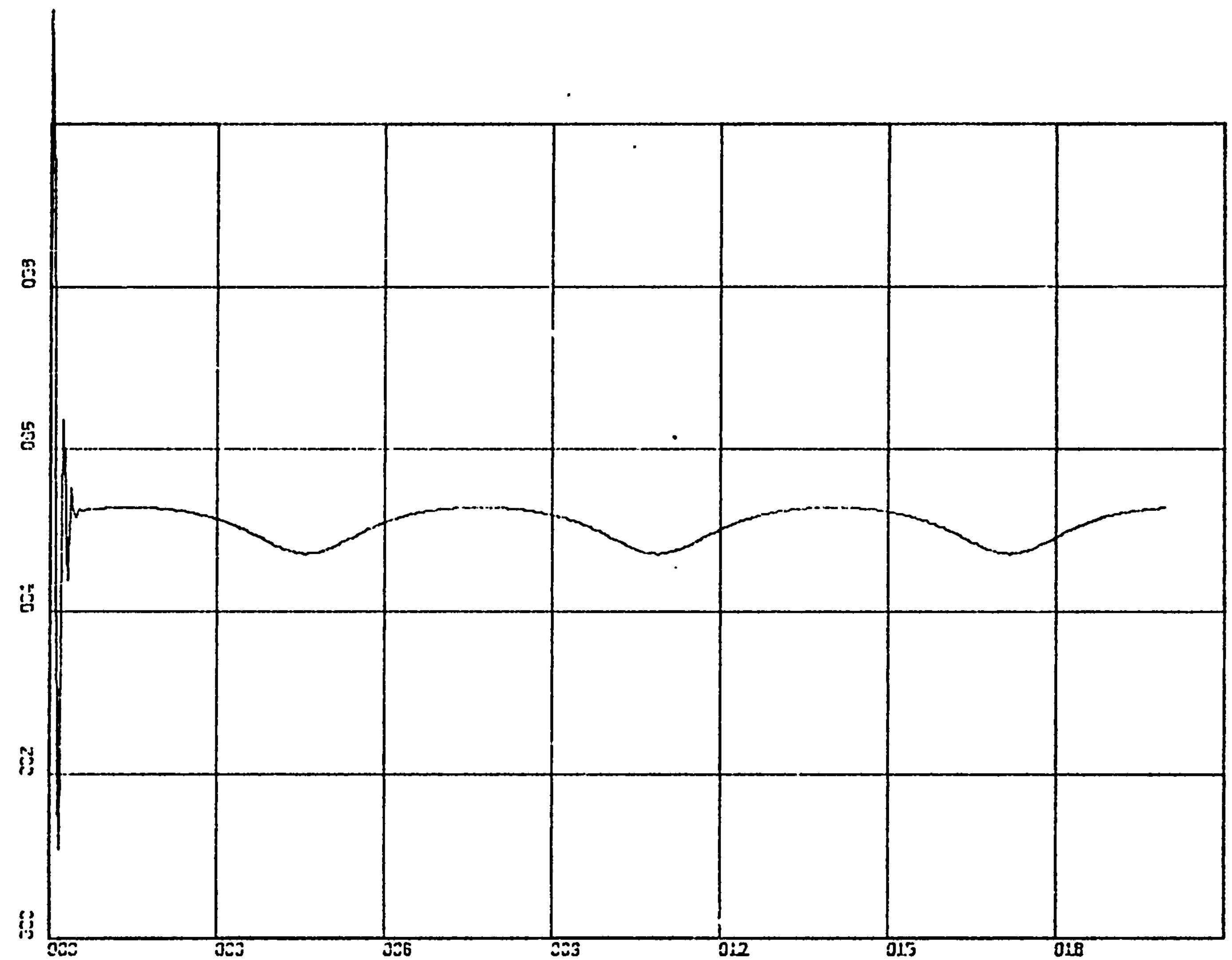


X: 3.0, Y: 2.0
Figure 67. Plot of Vo, Compensated System, Vin = 0.25.



X: 3.0, Y: 2.0

Figure 68. Plot of Vo, Compensated system, Vin = 0.45.



X: 3.0, Y: 2.0

Figure 69. Plot of Vo, Compensated System, Input Waveform Sinusoidal.

APPENDIX F

EXAMPLE 1 - GAIN CHARACTERISTIC FOR PROPORTIONAL VARIATIONS BETWEEN INPUT AND OUTPUT

Consider the AGC loop the block diagram of which is given in Appendix A, Fig. 28 and suppose the input amplitude varies sinusoidally from 1 to 10 volts or 20 db. The specifications of the loop are to keep the output amplitude extremes 1 db apart with minimum amplitude of 100 volts and the decibel variations of the output to be proportional to the decibel variations of the input, i.e. each instant of time

$$\log \frac{Vin}{Vinmin} = \log \frac{Vo}{Vomin} \text{ or } Vin(db) = Vo(db) - 40(db)$$

Having assigned the above relations the following can be calculated.

$$Vomax = 110 \text{ volts} \qquad \Delta Vo = 10$$

The curve (E) of Fig. 24 and 25 is going to be used as the gain characteristic.

Suppose two values of gain are chosen differing by 19 db

$$Nmax = 1.0, \qquad Nmin = .11$$

generating a $\Delta v = .49$ as determined from curve E.

Now in order to have practical values of gain the ordinate is multiplied by 100 and the abscissa by 10 thus

$$Nmax = 100.0, \qquad Nmin = 11.0$$

$$\Delta v = 4.9 \quad \text{and} \quad G_2 = \frac{\Delta v}{\Delta Vo} = \frac{4.9}{10} = .49$$

In this case (Vinmin)(Nmax) = Vomin, so $G_1 = 1.0$. Applying the formula derived in Chapter III

$$V_{REF} = \text{Vomin } G_2 + \text{vmin} = 100 \times .49 = 49.0$$

since vmin = 0.

Now, if the input waveform is a sinusoid of frequency 10 rad/sec the feedback low-pass filter must be chosen such that 10 rad/sec is in its passband. Suppose the feedback filter has the form $\frac{1}{.001s+1}$.

The derived numbers were tried in a simulation and the wave form of the output is shown in Fig. 70.

In order to show how the figures are interrelated around the loop the same example was again calculated with desired output amplitudes Vomax = 55, Vomin = 50 differing b

In this case

$$\text{Vinmin} \cdot \text{Nmax} \cdot G_1 = \text{Vomin}$$

thus

$$G_1 = \frac{\text{Vomin}}{\text{Vinmin} \cdot \text{Nmax}} = \frac{50}{(1)(100)} = .5$$

Now

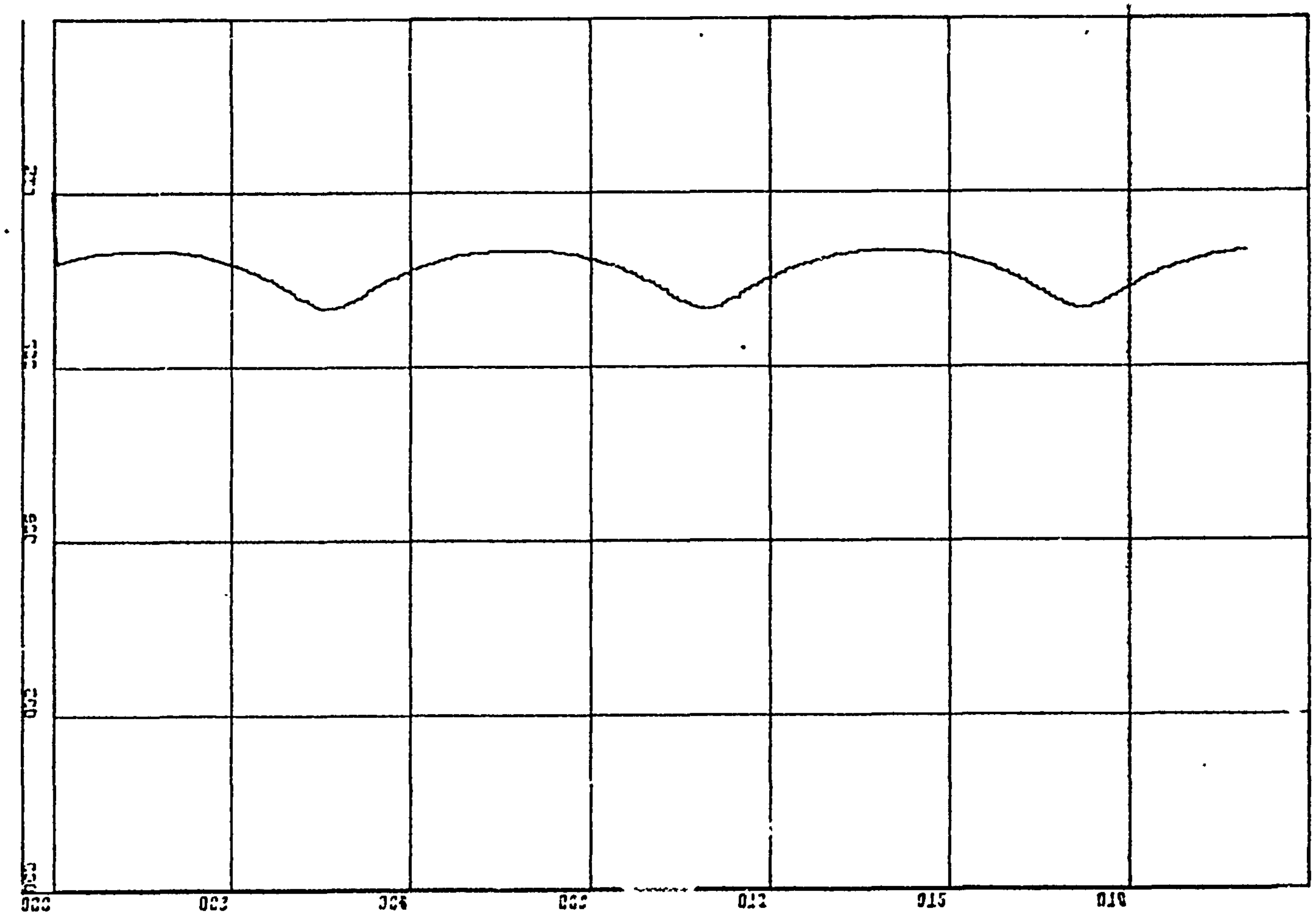
$$\Delta Vo = 5, \qquad \Delta v = 4.5$$

therefore

$$G_2 = \frac{\Delta v}{\Delta Vo} = \frac{4.5}{5} = .98$$

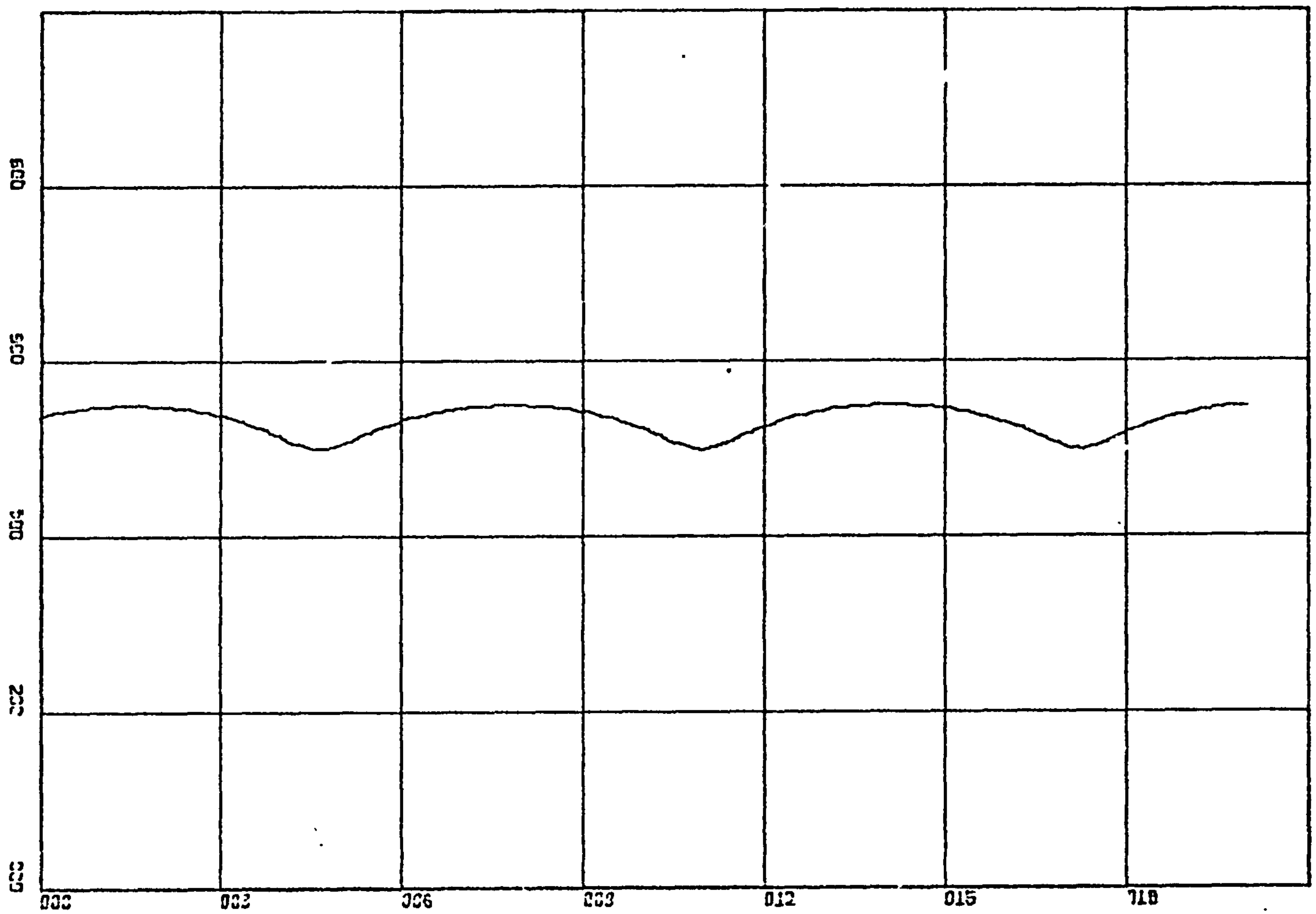
V_{REF} remains the same.

The resulting output waveform is shown in Fig. 71, where it is seen that it has the same shape as the previous one derived but the limits are lower according to the new specifications.



X: .3, Y: 3.0

Figure 70. Plot of Vo, Gain Characteristic Producing Linearity Between Input and Output When Both Are Expressed in Decibels, Vo = 100 - 110.



X; .3, Y: 2.0

Figure 71. Plot of Vo, Gain Characteristic Producing Linearity Between Input and Output When Both Are Expressed in Decibels, Vo = 50 - 55.

In both of the above simulations curve (E) was used storing points of the graph in a digital computer arbitrary function generator, and only the part of the graph required for each case was used.

EXAMPLE 2 - DISTORTIONLESS GAIN CHARACTERISTIC

Consider the AGC loop the block diagram of which is shown in Appendix A, Fig. 28 and suppose the input is a sine wave whose amplitude varies from 1 to 10 volts or 20 db. The specification of the loop is to produce a sinusoidal output with extreme values Vomax = 5.3, Vomin = 4.7 or 1 db apart.

The equation of the gain versus bias voltage v is the one given by equation (4-24) multiplied by a factor of 100 in order to make the gain values used practical

$$N(v) = 100 \times \frac{V+v}{V+10^5 v}$$

Choosing two values of gain separated by 19 db, Nmax = 89.0, Nmin = 10.0, the following can be calculated making use of equation (4-25) with normalized values of gain.

$$vmin = \frac{(1-.89)V}{(.89)(10^5)-1} \simeq \frac{.11V}{89000} = 1.235 \times 10^{-6} V$$

$$vmax = \frac{}{(.1)(10^5)-1} \simeq \frac{.9V}{10000} = 9 \times 10^{-5} V$$

$$\Delta v = 8.8765 \times 10^{-5} V$$

Assign arbitrarily V = 10^5. Then

$$\Delta v = 8.88$$

Since

$$\Delta Vo = .6$$

$$G_2 = \frac{8.88}{.6} = 14.8$$

and

$$G_1 = \frac{4.7}{89.0} = .0529$$

$$V_{REF} = Vomin\ G_2 + vmin = 14.8 \times 4.7 - .123 = 69.377$$

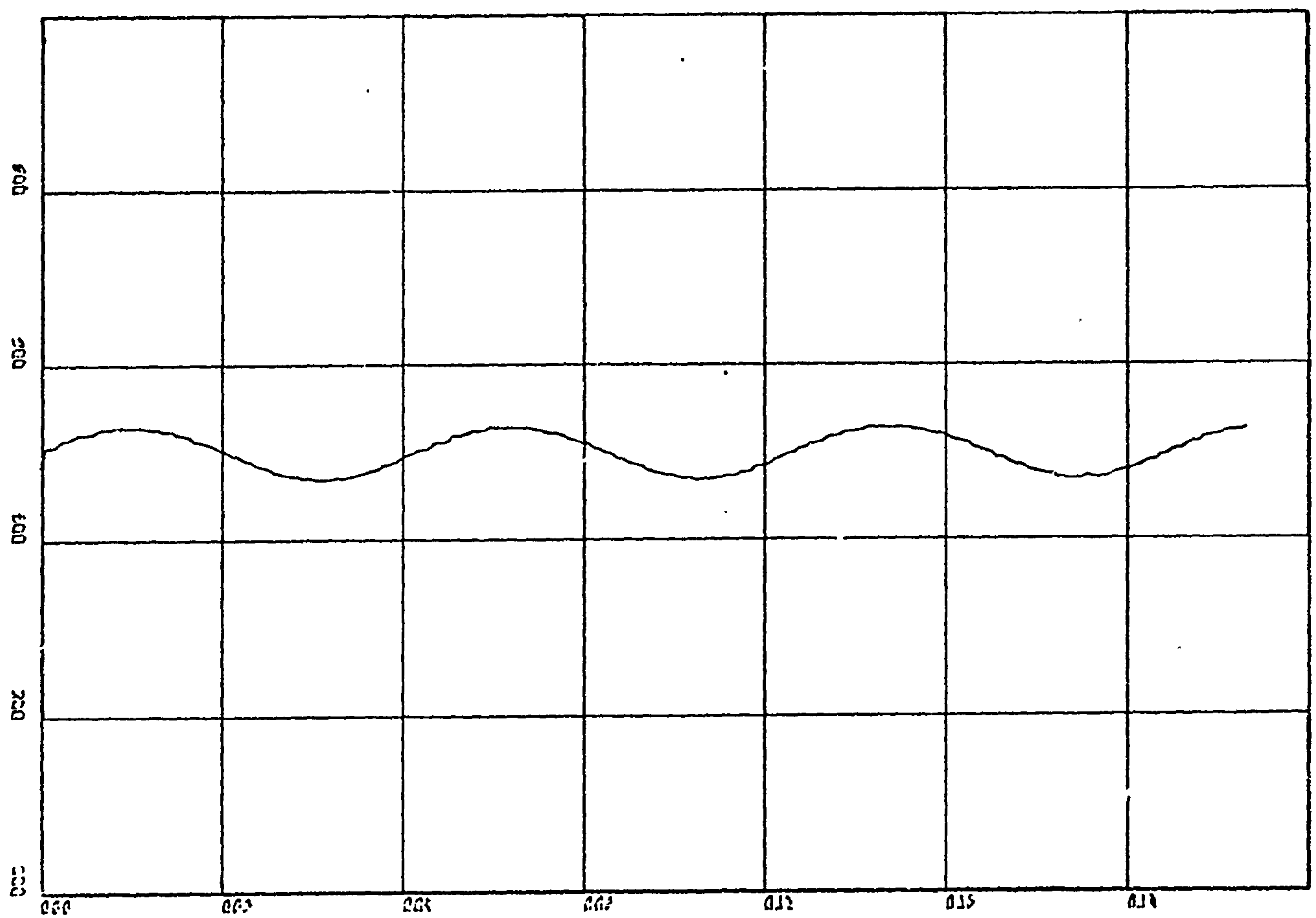
Finally assuming the frequency of the input sinusoid 10 rad/sec, one can assign the feedback filter as $\frac{1}{.001s+1}$

The resulting output waveform of this simulation is shown in Fig. 72 where it is seen that both the specifications of the magnitude and linearity have been met.

In the same way as the above a second example was constructed suppressing 30 db of input amplitude variations (Vinmin = 1.0, Vinmax = 31.6) to 1 db of output amplitude variations (Vomin = 4.7, Vomax = 5.3) and the result identical to the previous one is shown in Fig. 73.

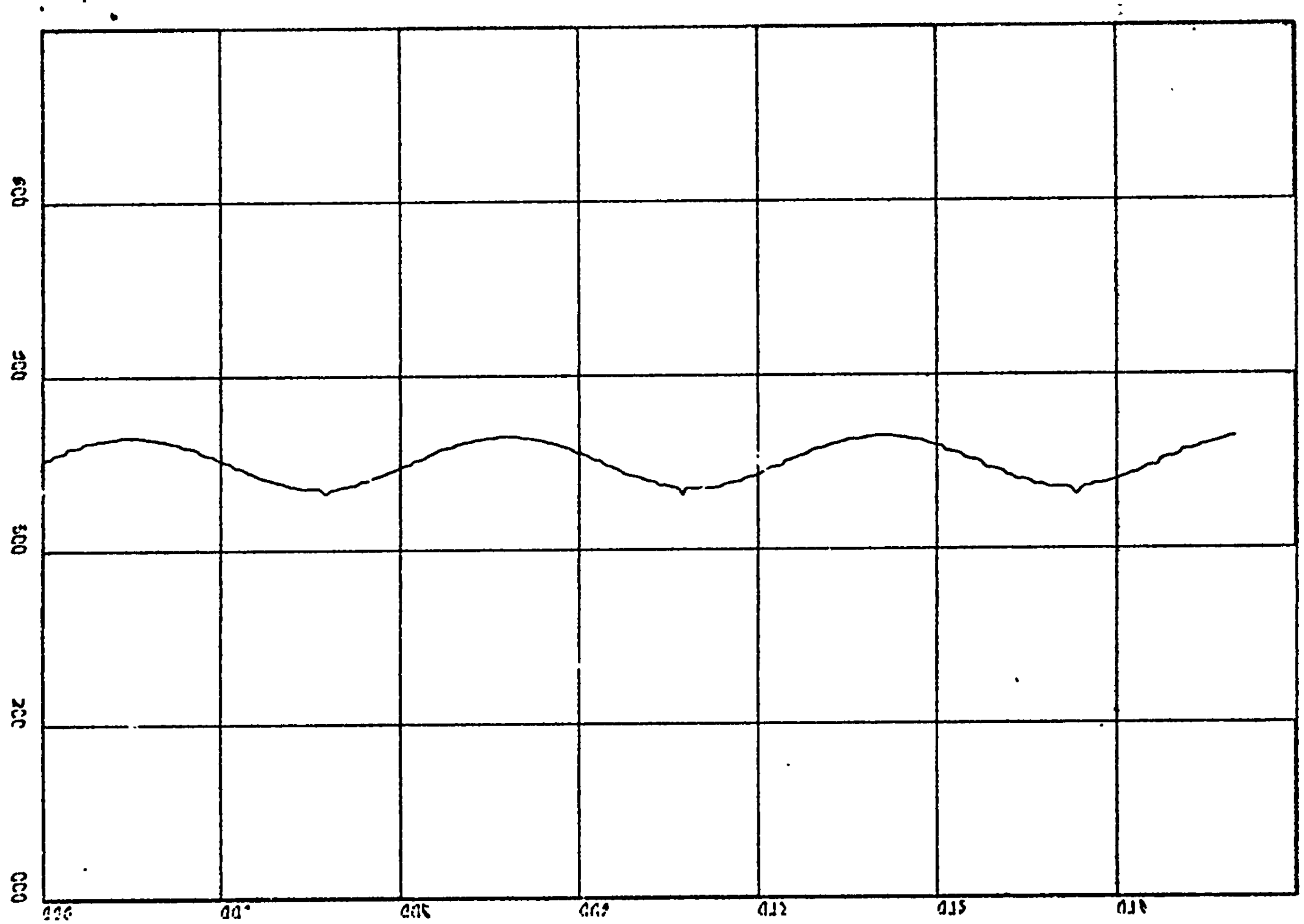
Finally 40 db of input amplitude variations (Vinmin = 1.0, Vinmax = 100.0) to 1 db of output amplitude variations (Vomin = 47.0, Vomax = 53.0) were suppressed resulting in the output waveform shown in Fig. 74.

As a further check of the developed theory a simulation was tried identical to the first example suppressing 20 db into 1 but this time the input waveform was a triangular one of frequency 10 rad/sec.



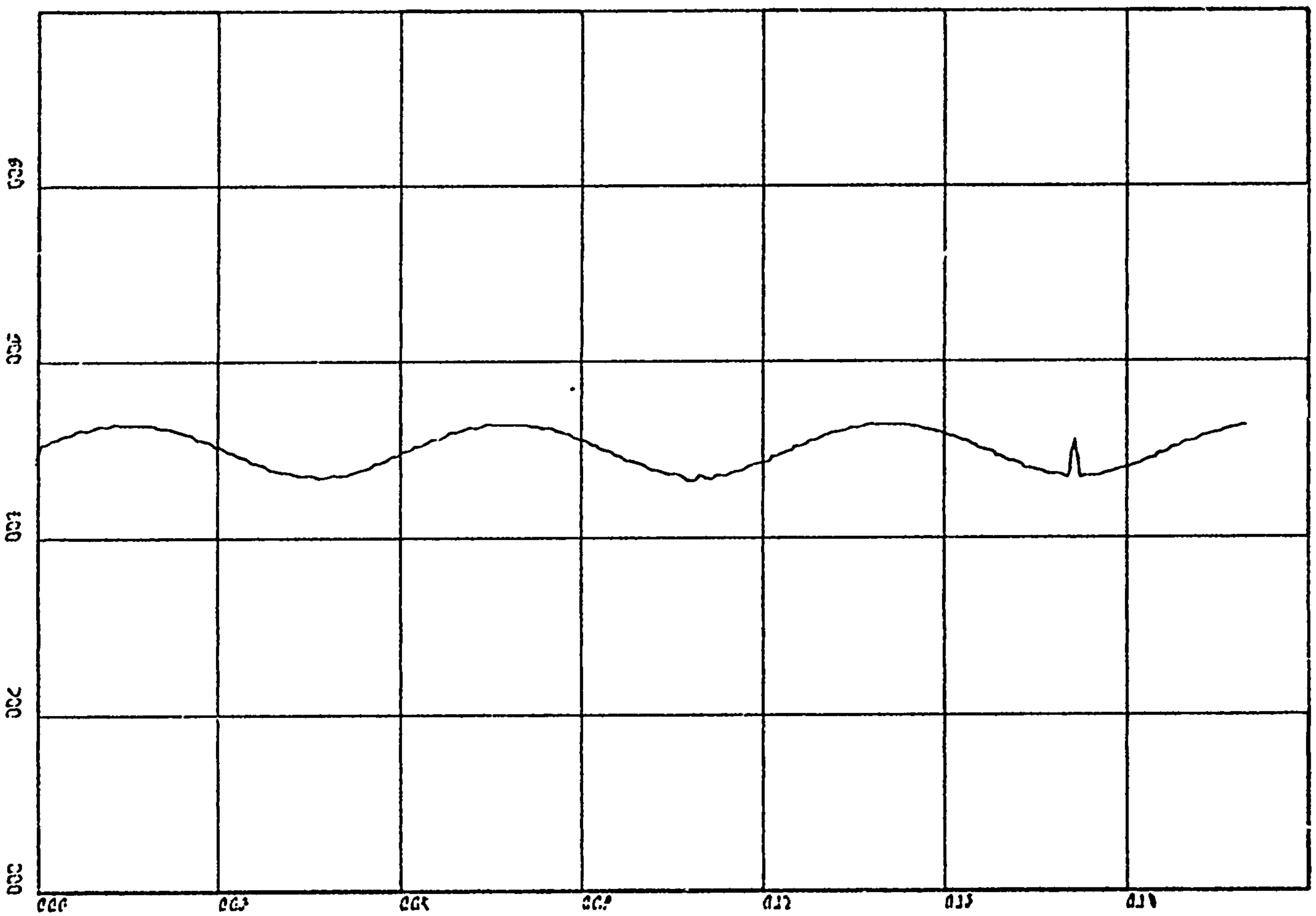
X: .3, Y: 2.0

Figure 72. Distortionless Characteristic, Regulation of 20 db to 1 db.



X: .3, Y: 2.0

Figure 72. Distortionless Characteristic, Regulation of 30 db to 1 db.



X: .3, Y: 2.0

Figure 74. Distortionless Characteristic, Regulation of 40 db to 1 db.

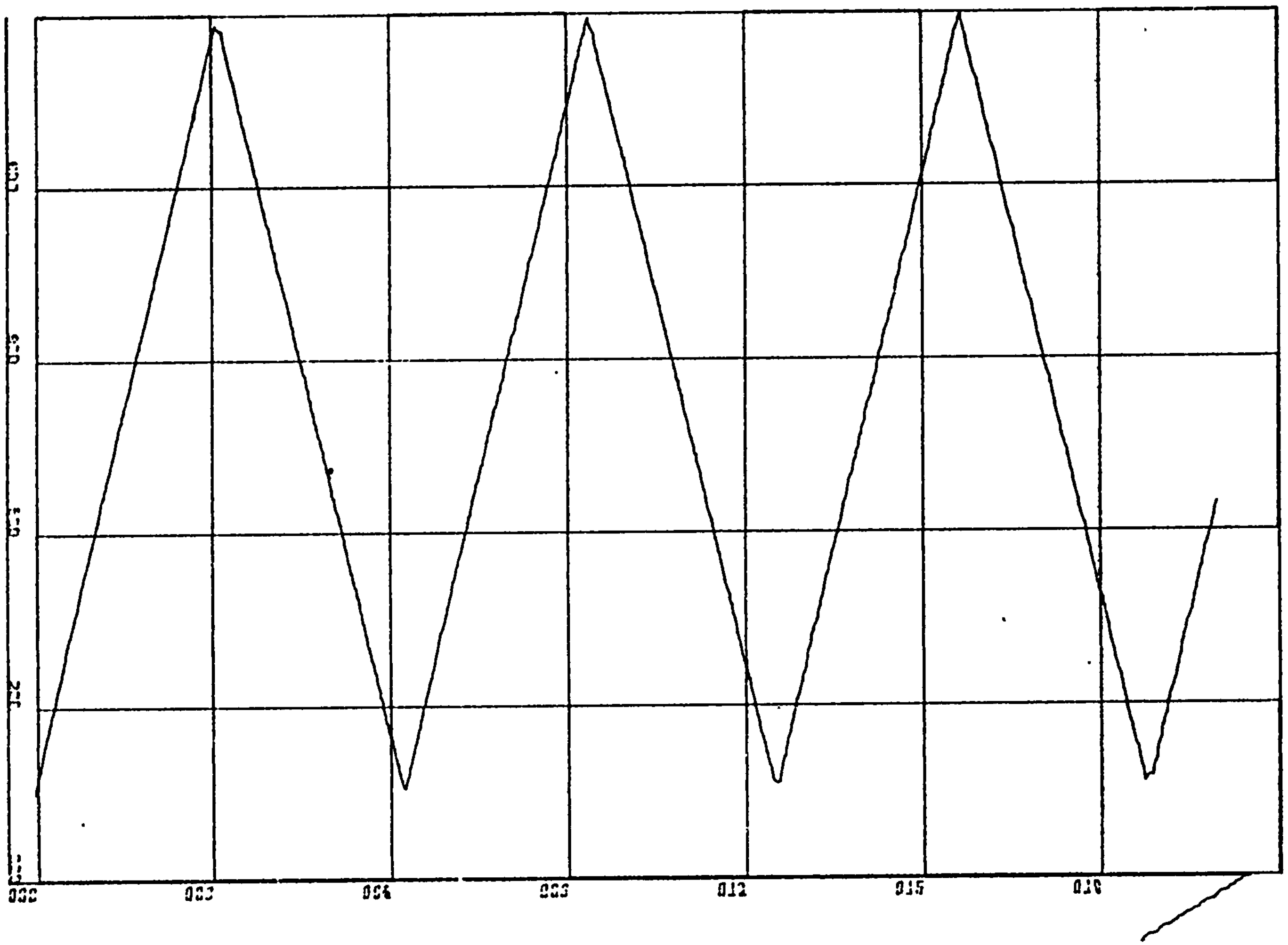
Since a pole at 1,000 was provided in the feedback path it was expected that most of the Fourier components of the input waveform would pass the pole and sum up to constitute the triangular waveform at the output. The components outside the passband of the feedback filter would be of very low amplitude not affecting the output waveform.

Figures 75 and 76 show the input and output waveforms plotted in the same scale and comparison shows the regulation of the AGC action and how the waveform shape was preserved because of the non-linear gain characteristic.

EXAMPLE 3 - DISTORTIONLESS CHARACTERISTIC - PHASE SHIFT

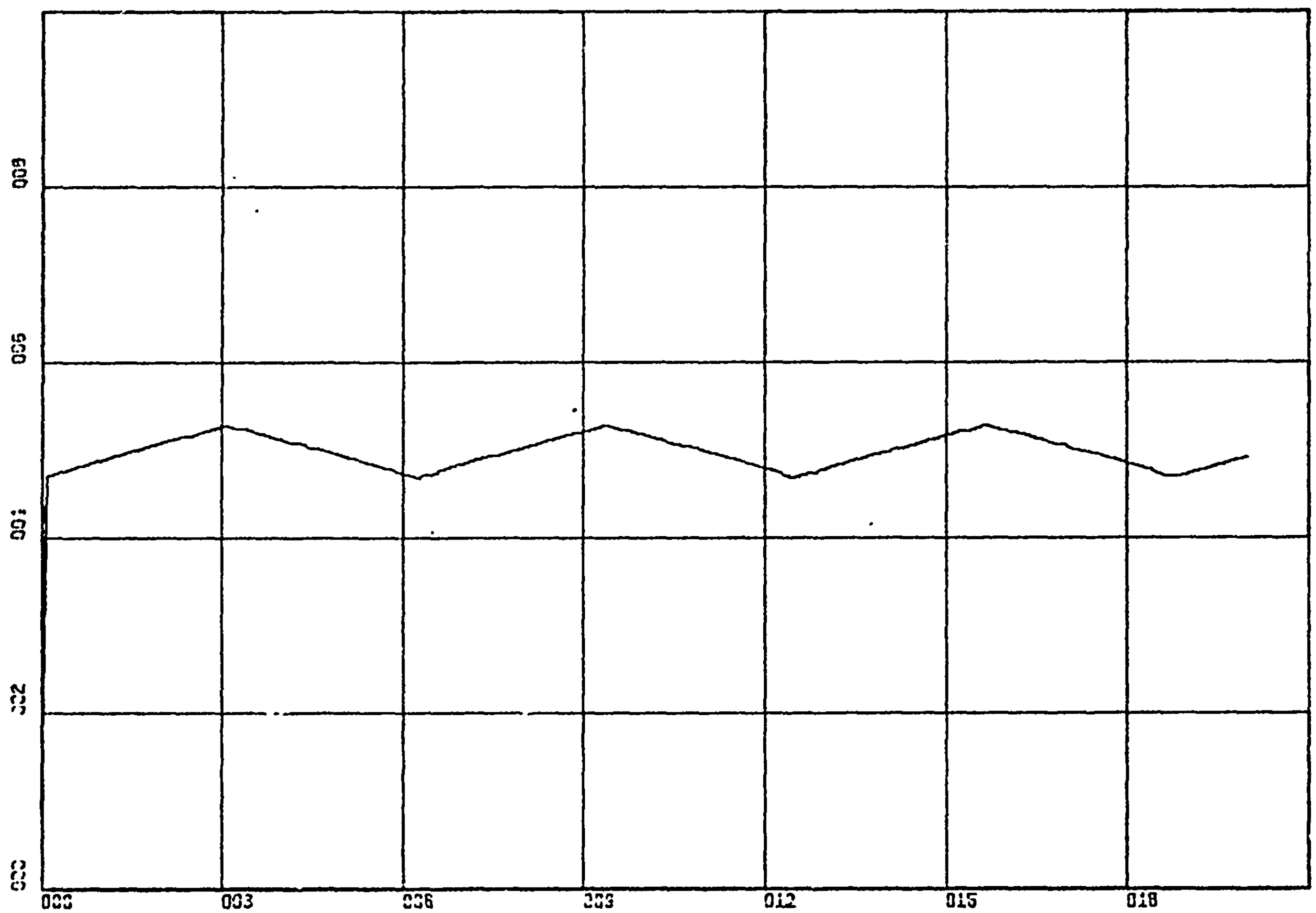
The phase shift produced by the particular loop was checked using a sinusoidal waveform as input. The suppression in this example and all the following was adjusted to 20 db to 1 db. Using a pole at 1000 in the feedback filter and a frequency of input waveform 10 rad/sec no appreciable phase shift could be produced. Additional runs were tried with feedback poles at 1000, 100, 20, 10, 5. The output waveforms obtained are shown in Figures 77, 78, 79, 80 and 81. In all cases the input waveform was a sine wave with phase shift of 0°. From the above figures the resulting phase shift is easily observed at t = 0, and it is more pronounced as the input frequency approaches the pole frequency.

The resulting phase shift is a leading one and this fact is explained if one considers a linear feedback loop with a pole in the feedback path then



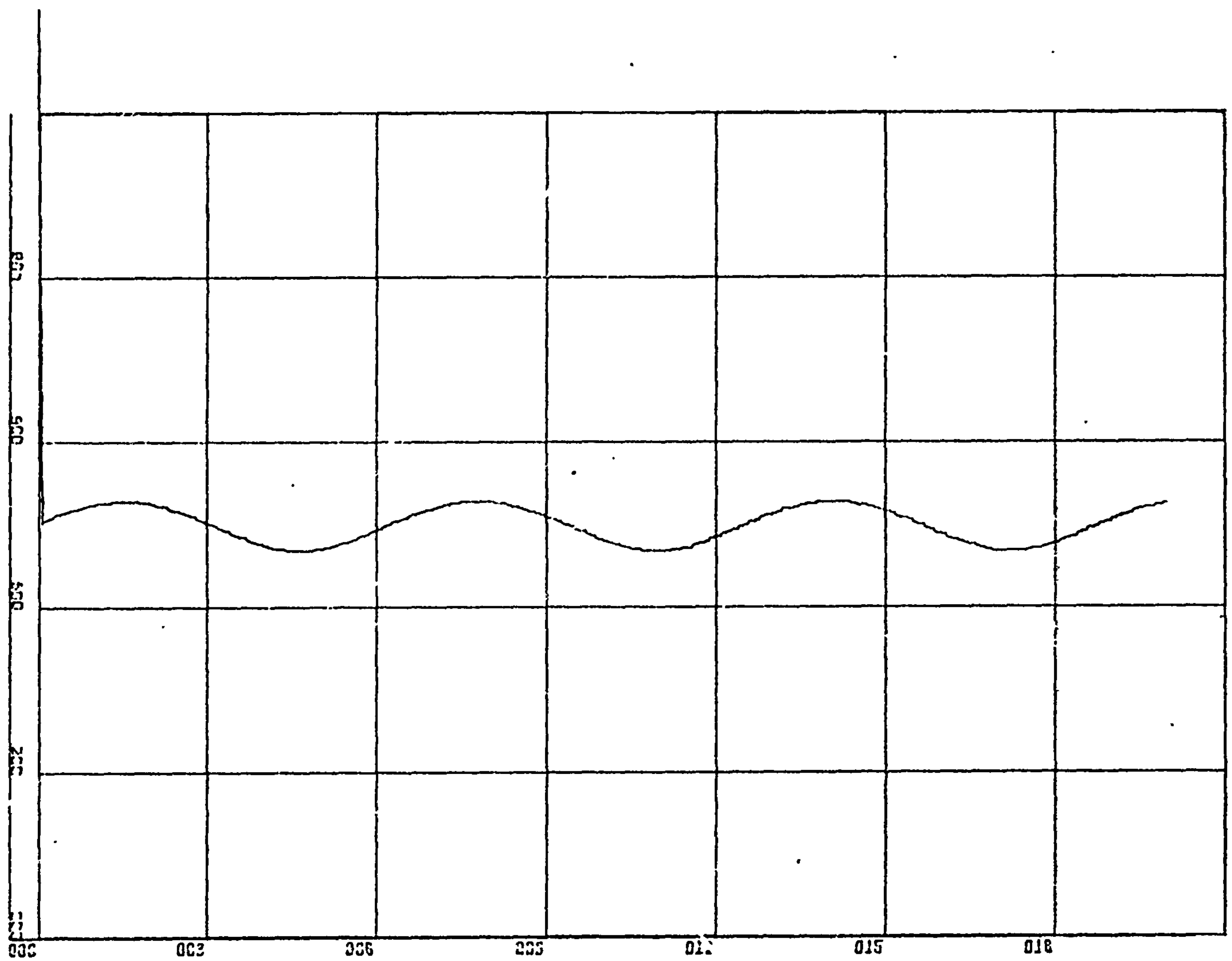
X: .3, Y: 2.0

Figure 75. Distortionless Characteristic, Input Waveform Applied to the Loop.



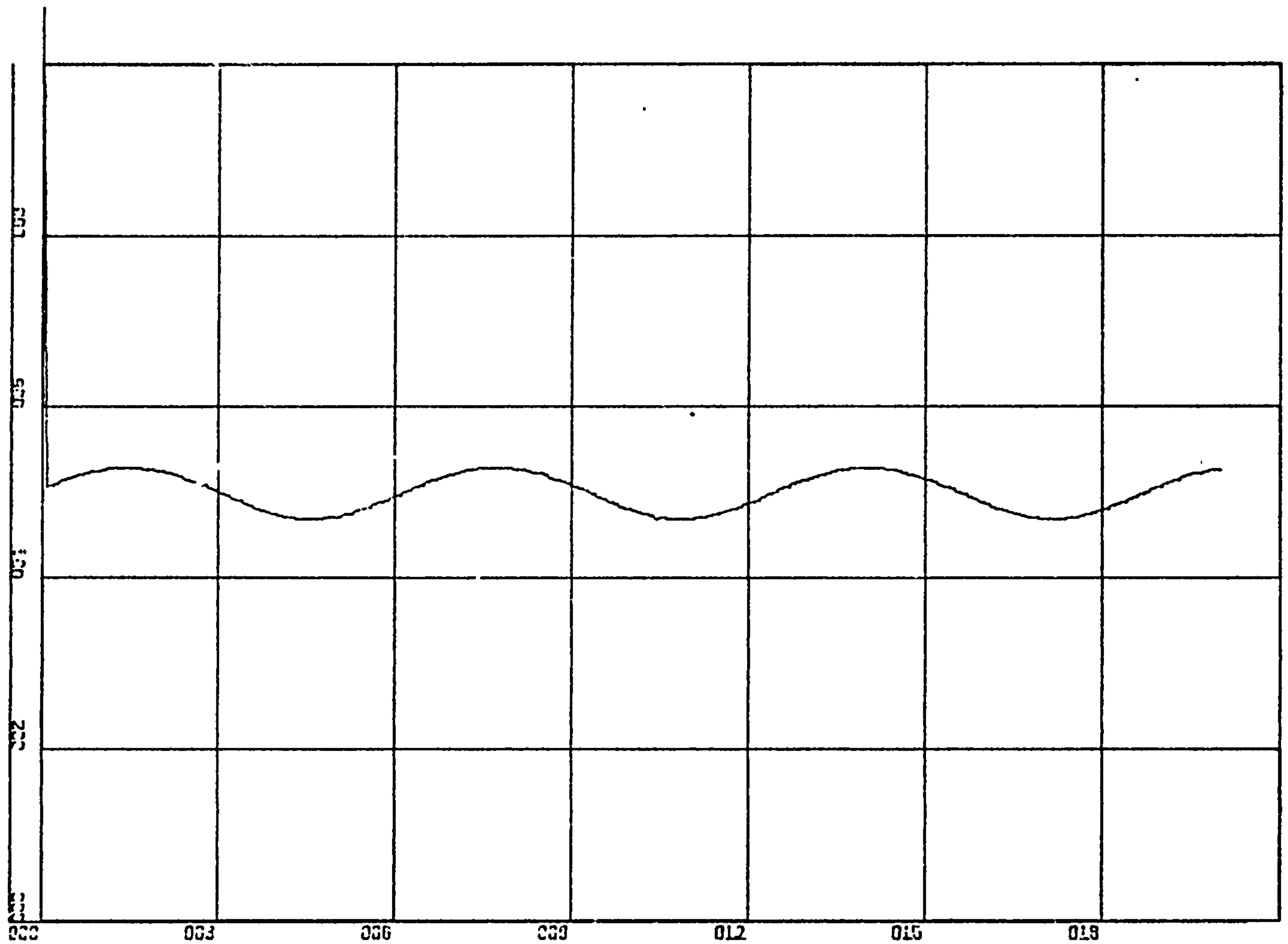
X: .3, Y: 2.0

Figure 76. Distortionless Characteristic, Output Waveform for Input Fig. 76.



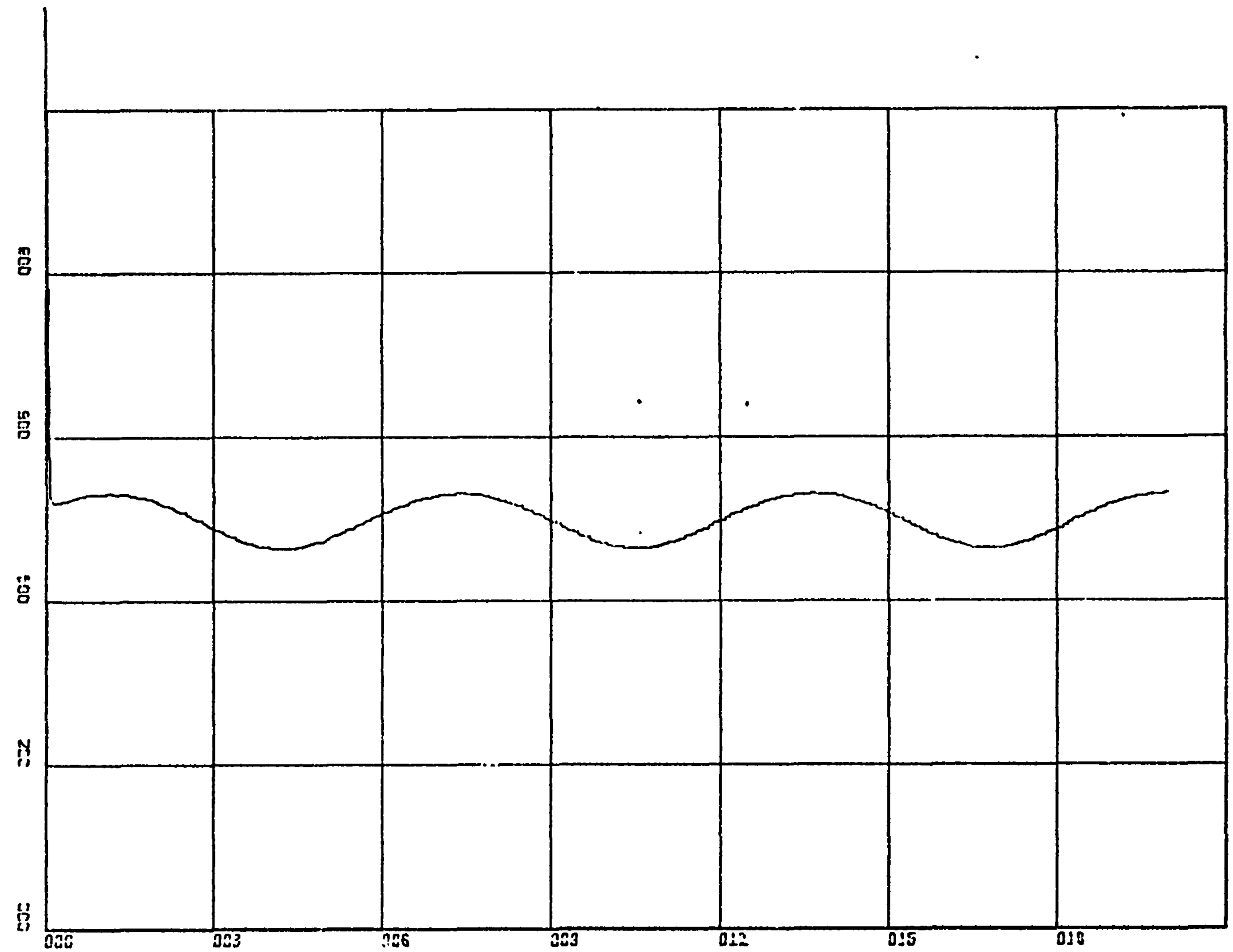
X: .3, Y: 2.0

Figure 77. Distortionless Characteristic, Phase Shift Pole at 1000 rad/sec.



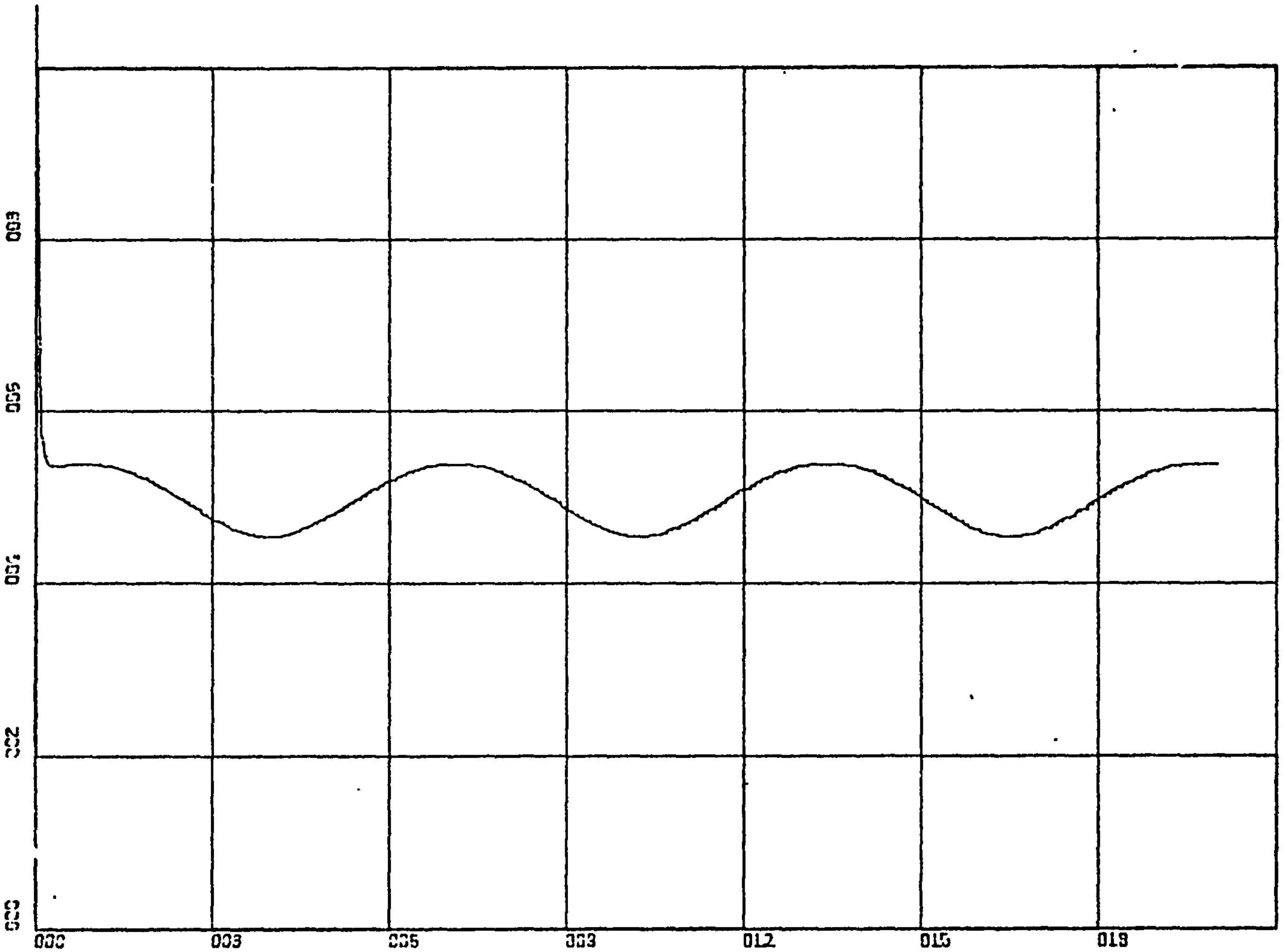
X: .3, Y: 2.0

Figure 78. Distortionless Characteristic, Phase Shift, Pole at 100 rad/sec.



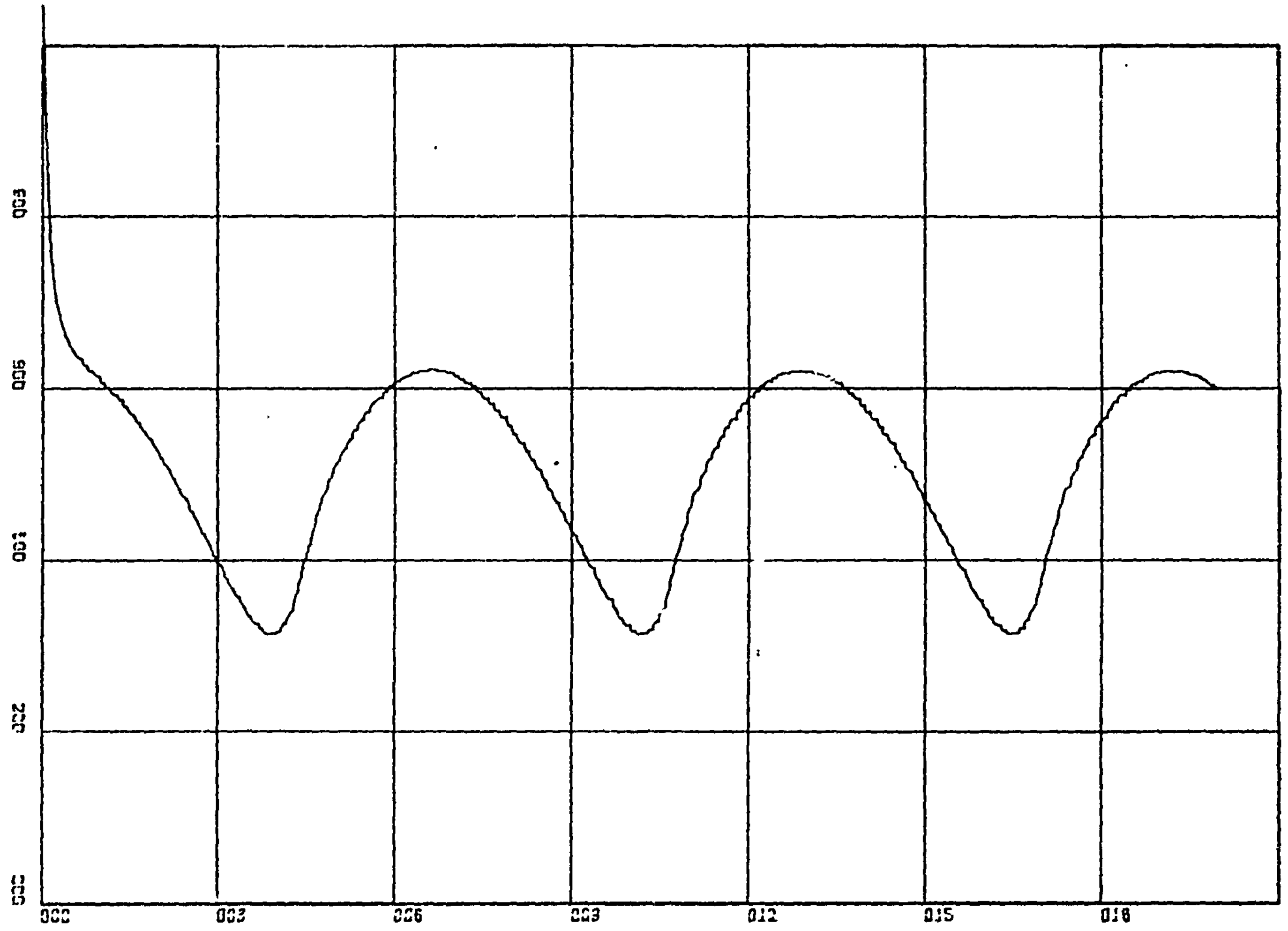
X: .3, Y: 2.0

Figure 79. Distortionless Characteristic, Phase Shift, Pole at 20 rad/sec.



X: .3, Y: 2.0

Figure 80. Distortionless Characteristic, Phase Shift, Pole at 10 rad/sec.



X: .3, Y: 2.0

Figure 81. Distortionless Characteristic, Phase Shift, Pole at 2 rad/sec.

$$\frac{C}{R} = \frac{K}{1 + \frac{K}{s+p}} = \frac{K(s+p)}{s+p+K} = \frac{Kp}{p+K} \cdot \frac{(\frac{1}{p}s+1)}{(\frac{1}{p+K}s+1)}$$

a zero and a pole are produced with the zero having a smaller value than the pole which is the definition of the "lead" filter.

Another effect which can be observed is that as the frequency approaches the value of the pole the regulation becomes slightly poorer and when the frequency becomes larger than the pole, distortion is generated which was expected according to what has been stated.

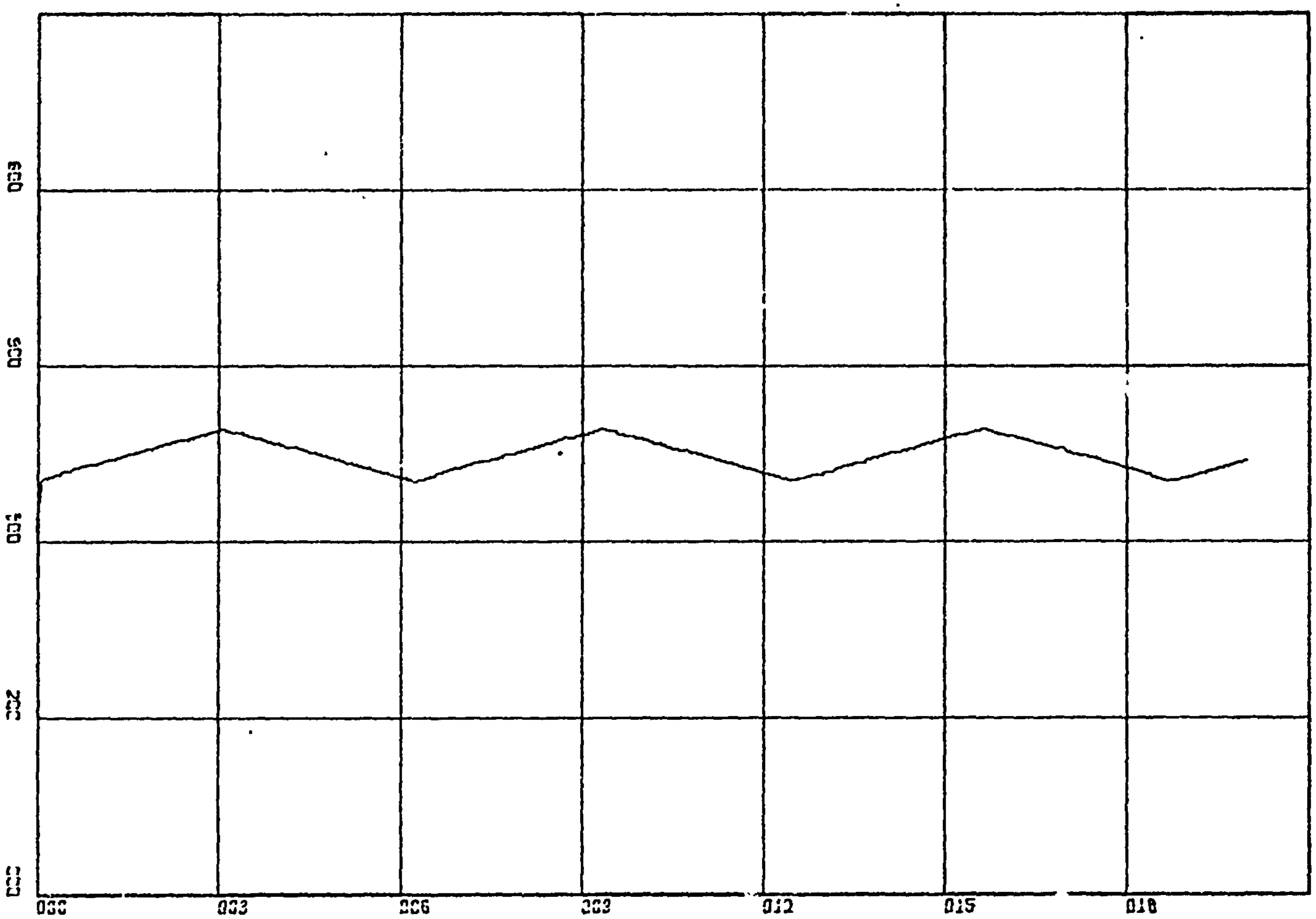
The same runs were tried with a triangular input waveform generating the output waveforms shown in Figures 82, 83, 84, 85 and 86.

The resulting distortion is clear as the input frequency approaches the value of the feedback pole.

EXAMPLE 4 - DISTORTIONLESS CHARACTERISTIC - TRANSIENT RESPONSE

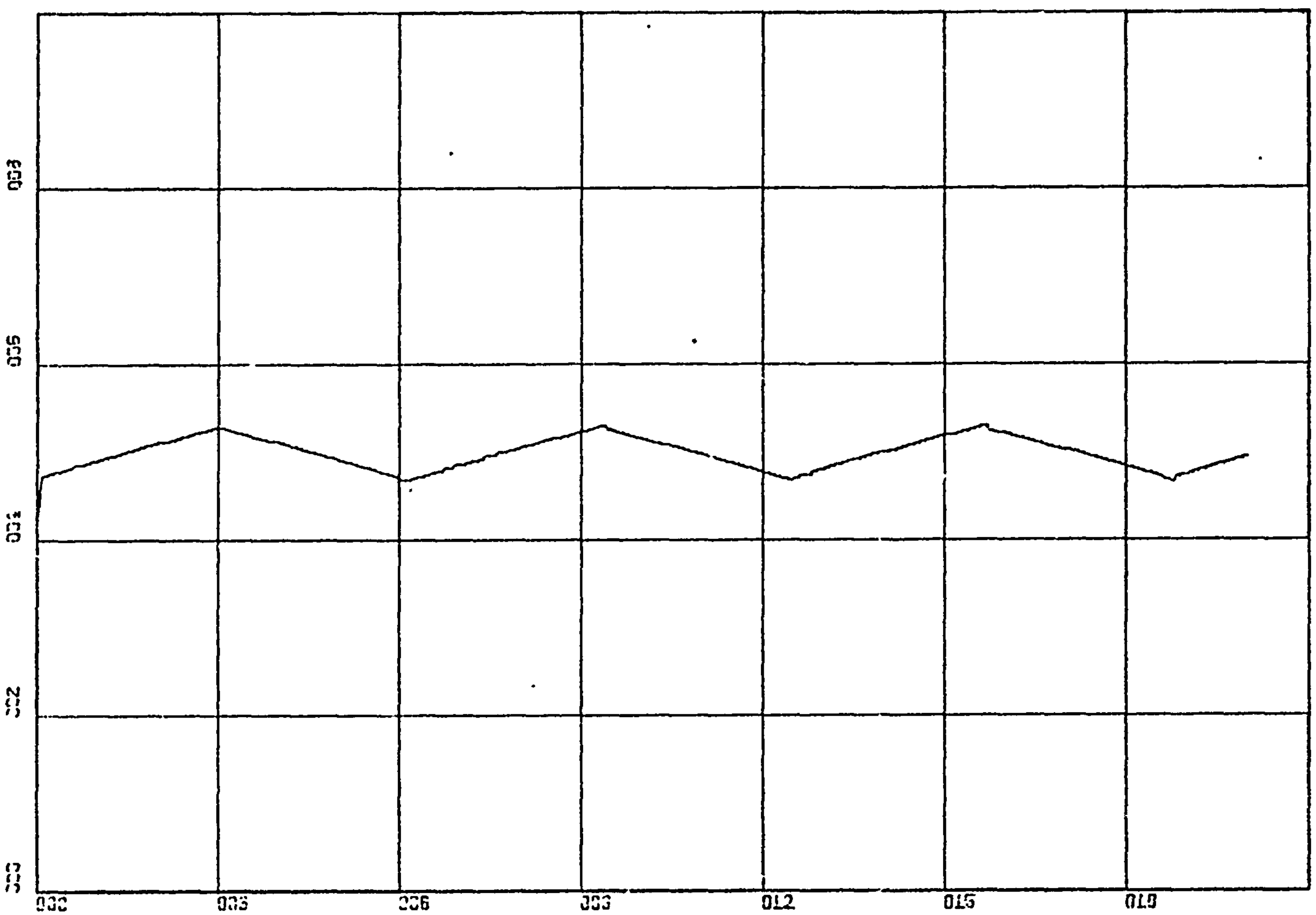
Finally the transient response was checked for the various values of the feedback pole setting with the input waveform a triangle.

At $t = 0$, since no bias voltage exists, the gain has its maximum value and the output is just the input multiplied by this maximum value of the gain thus generating the rising ramp. As soon as $Vo \cdot G_2$ becomes greater than V_{REF} a negative bias voltage exists and the value of the non-linear gain falls rapidly to values required by the AGC action and the output starts taking values between the prescribed limits. Now as the



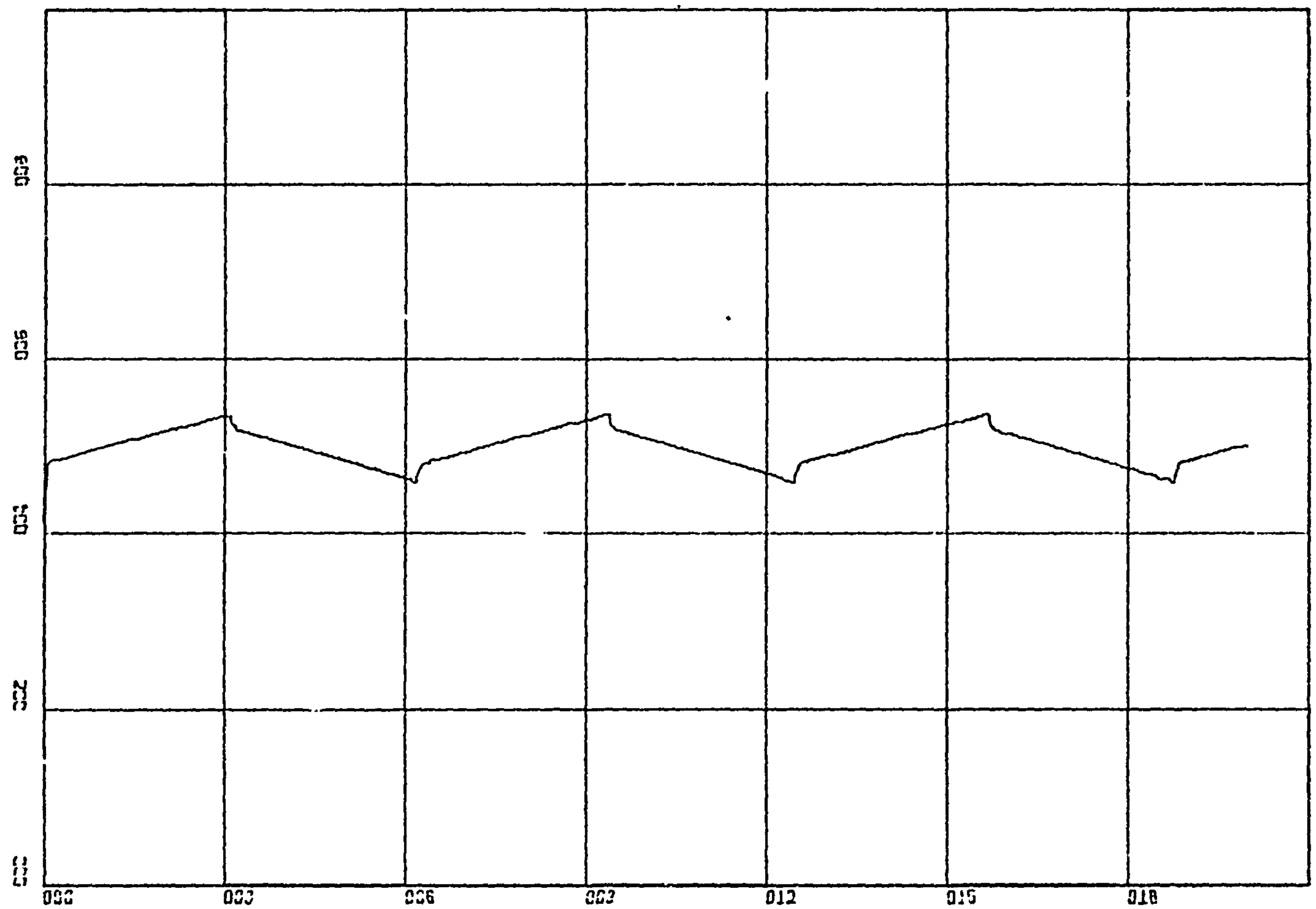
X: .3, Y: 2.0

Figure 82. Distortionless Characteristic, Phase Shift, Pole at 1000 rad/sec.



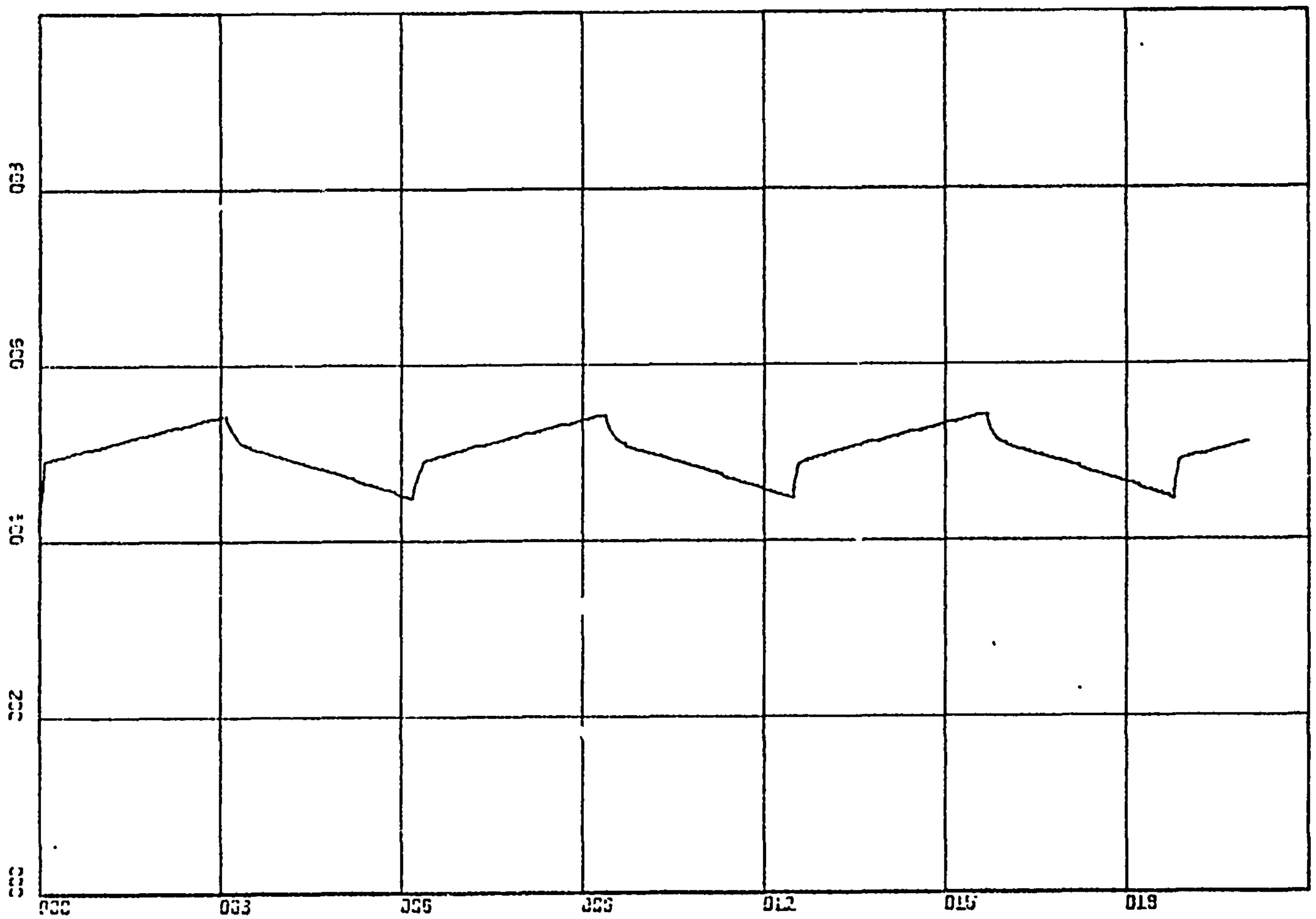
X: .3, Y: 2.0

Figure 83. Distortionless Characteristic, Phase Shift,
Pole at 100 rad/sec.



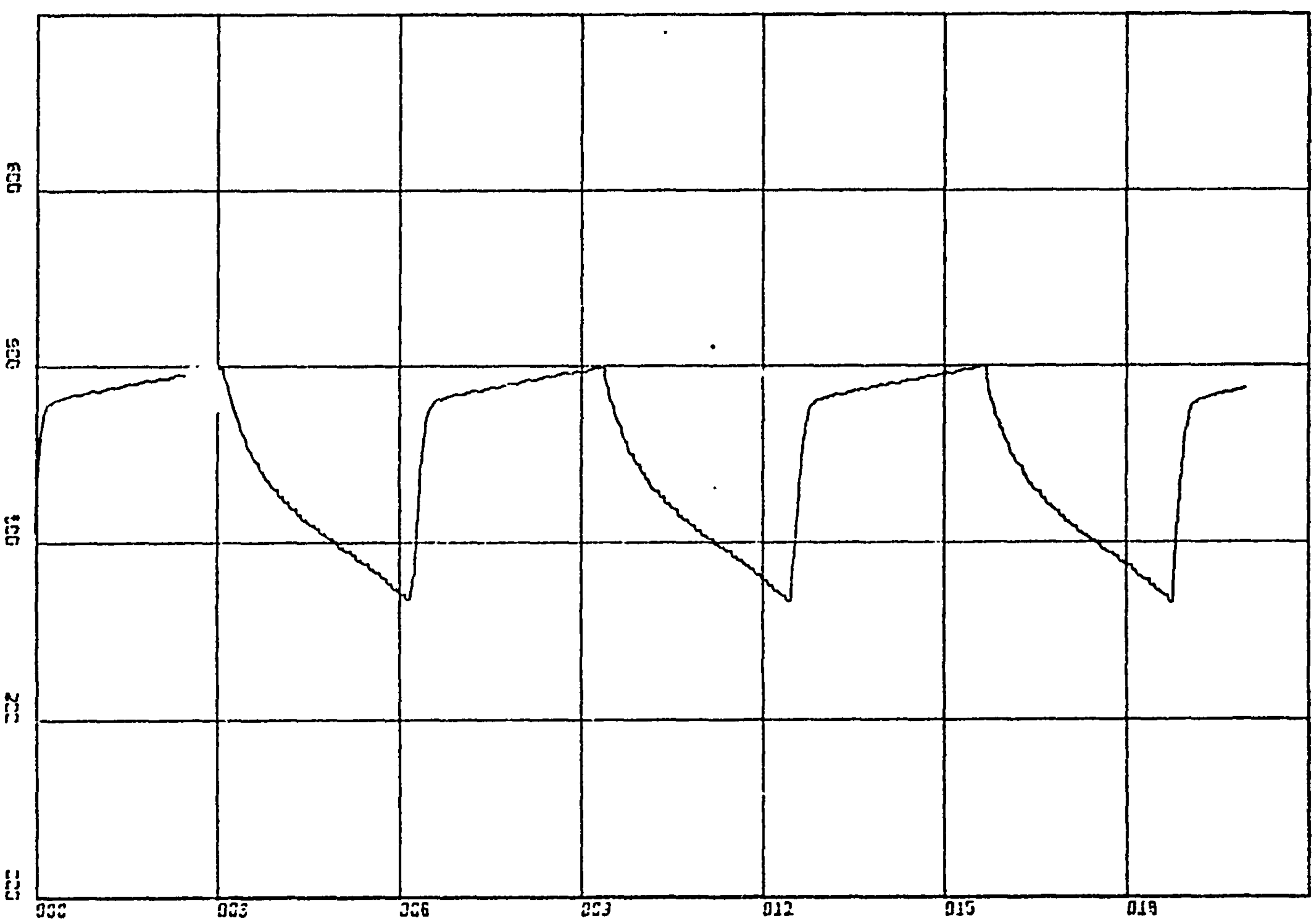
X: .3, Y: 2.0

Figure 84. Distortionless Characteristic, Phase Shift, Pole at 20 rad/sec.



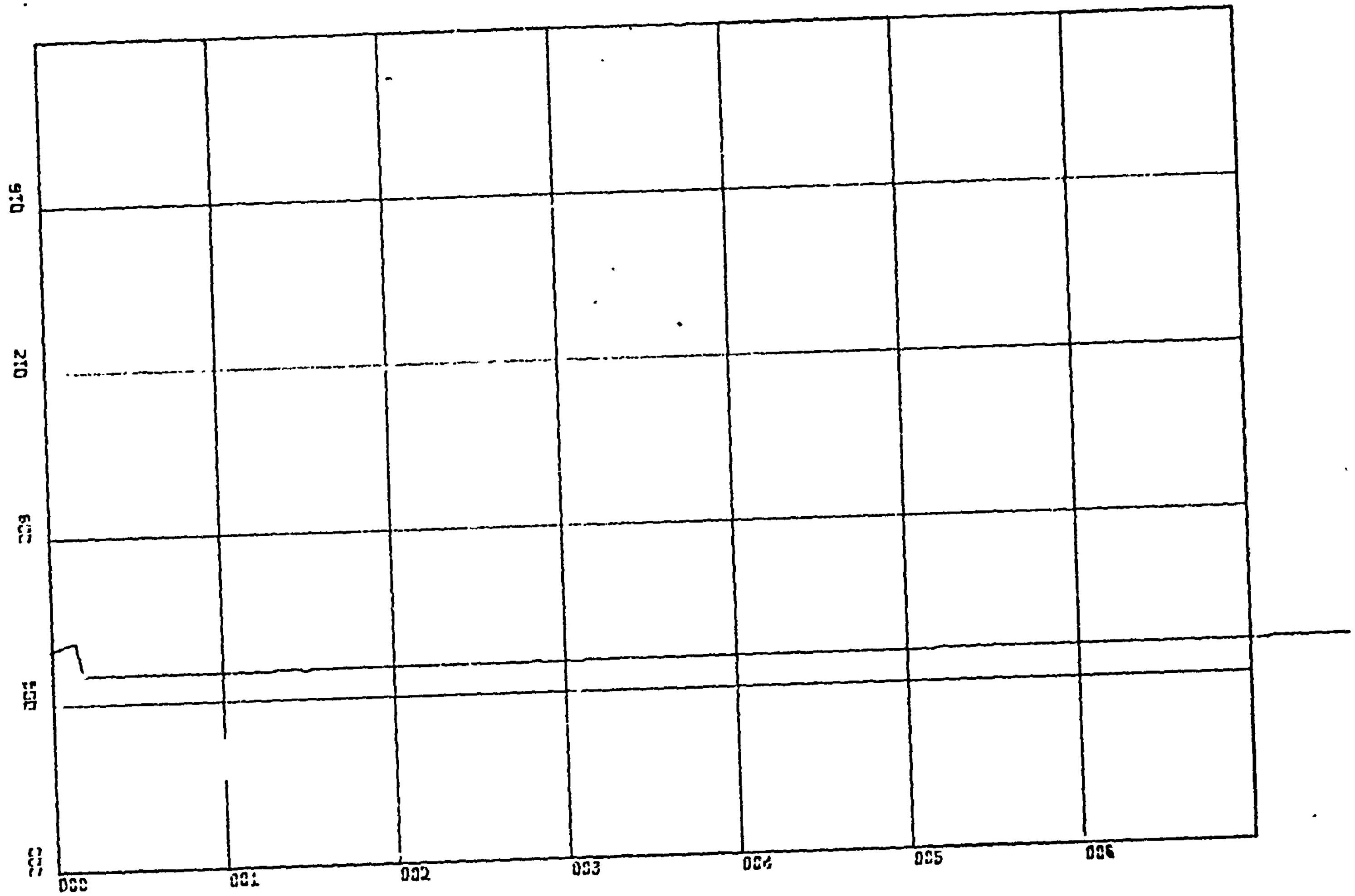
X: .3, Y: 2.0

Figure 85. Distortionless Characteristic, Phase Shift, Pole at 10 rad/sec.



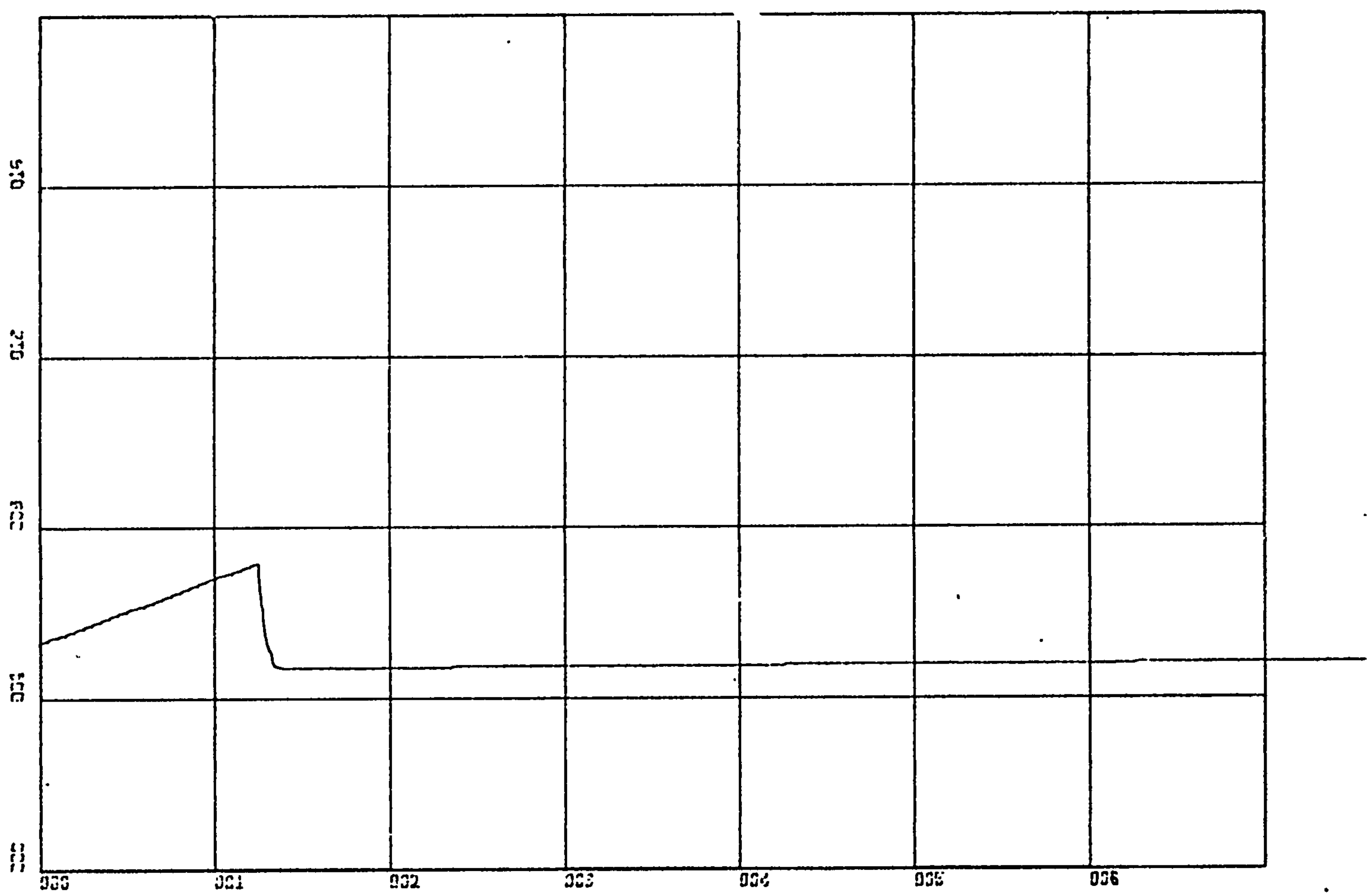
X: .3, Y: 2.0

Figure 86. Distortionless Characteristic, Phase Shift, Pole at 2 rad/sec.



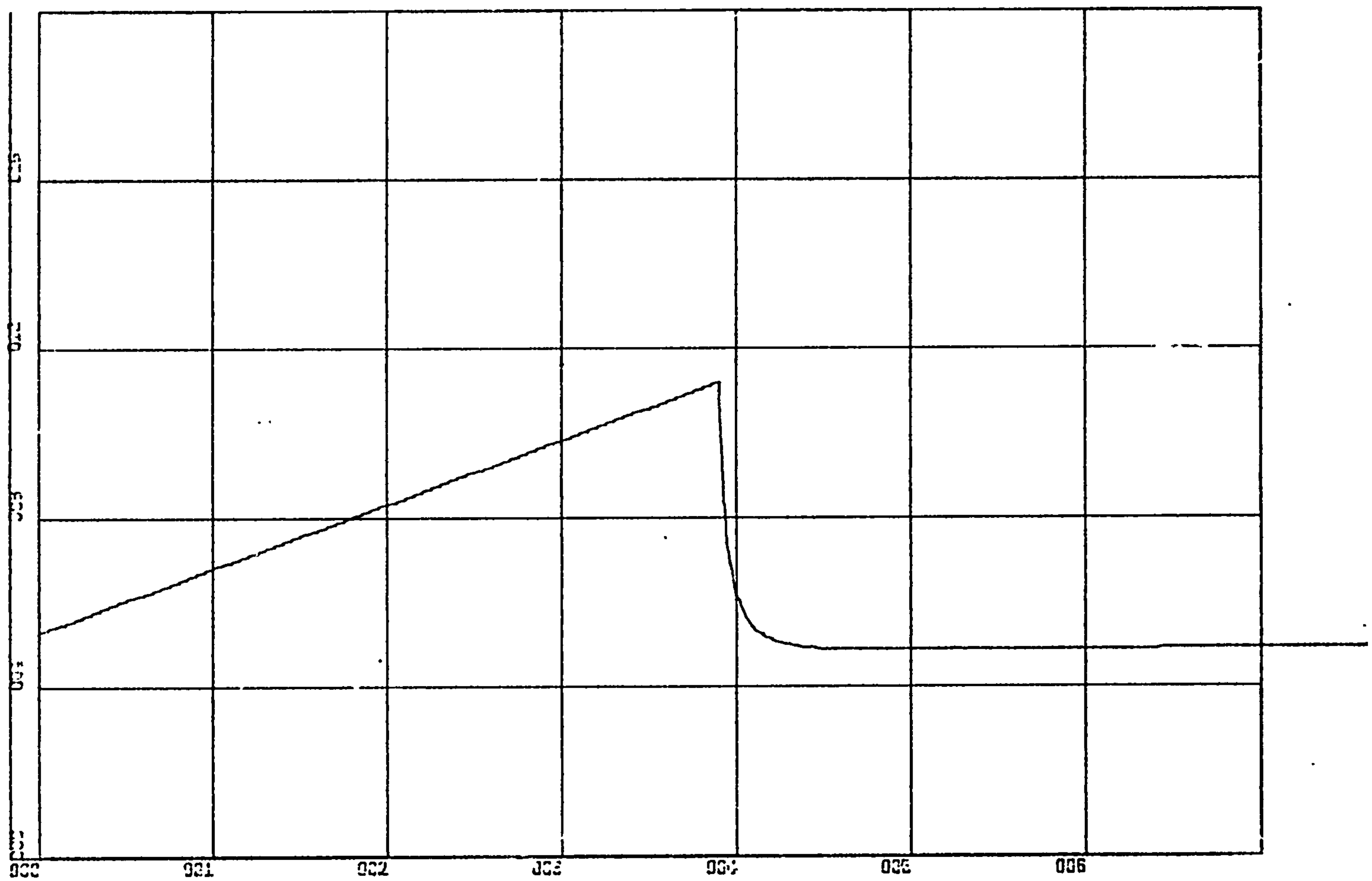
X: .01, Y: 4.0

Figure 87. Distortionless Characteristic Transient Response, Pole at 1000 rad/sec.



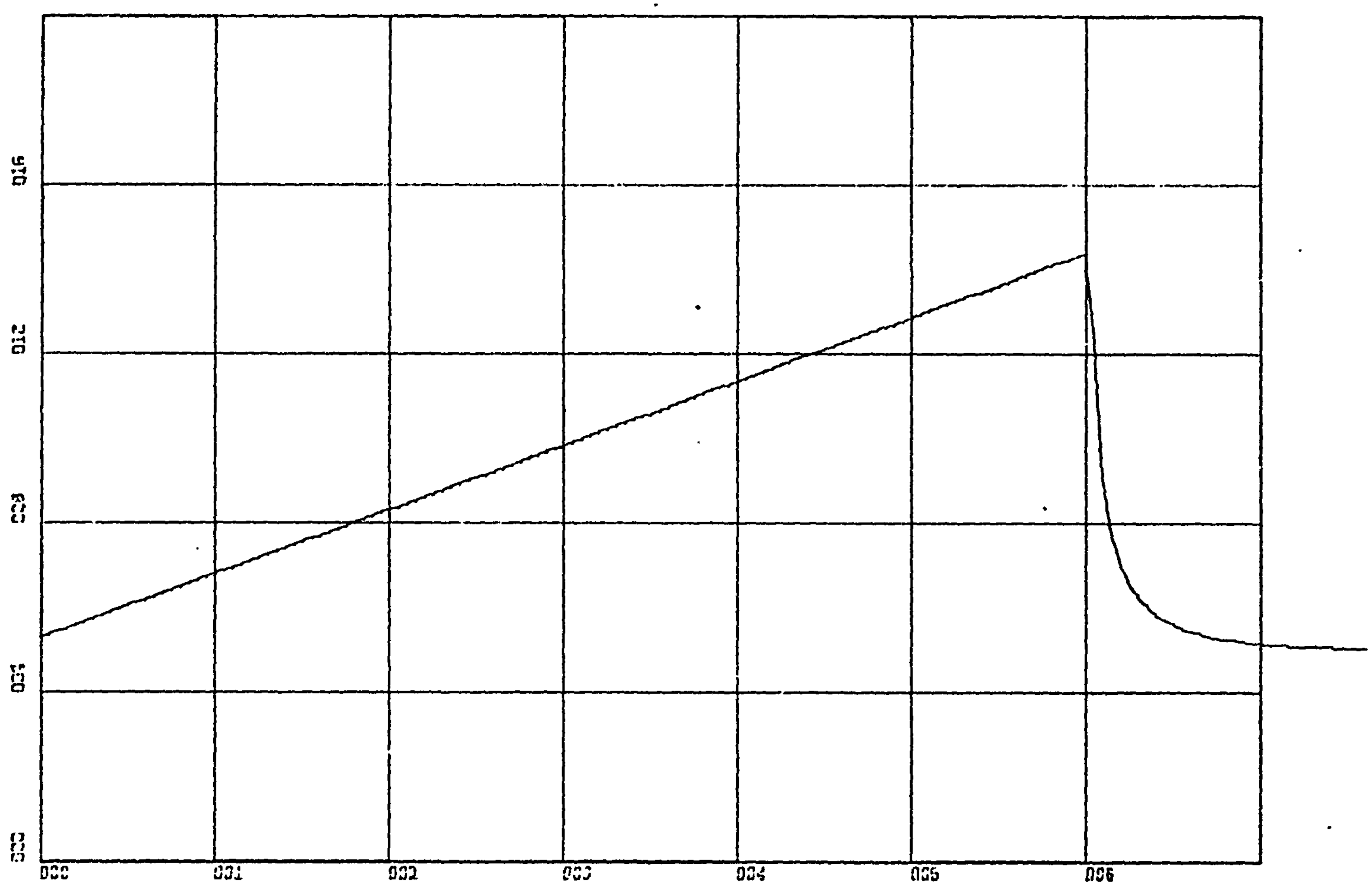
X: .01, Y: 4.0

Figure 88. Distortionless Characteristic, Transient Response, Pole at 100 rad/sec.



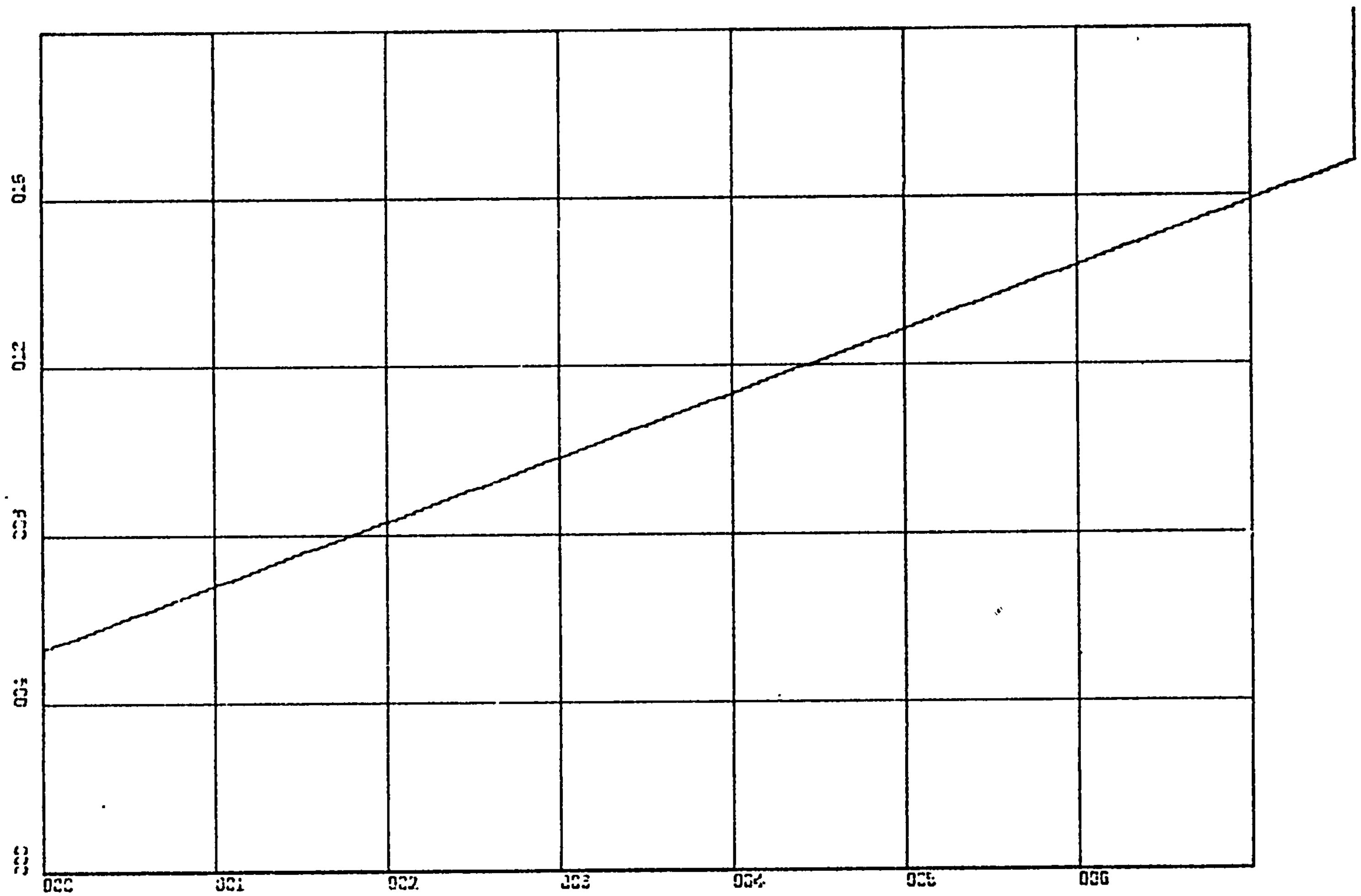
X: .01, Y: 4.0

Figure 89. Distortionless Characteristic, Transient Response, Pole at 20 rad/sec.



X: .01, Y: 4.0

Figure 90. Distortionless Characteristic, Transient Response, Pole at 10 rad/sec.



X: .01, Y: 4.0

Figure 91. Distortionless Characteristic, Transient Response, Pole at 2 rad/sec.

feedback pole becomes smaller in magnitude the time constant becomes bigger and the duration of the transient period becomes longer.

It is worthwhile noting again that all the simulations were attempted with a non-linear gain characteristic given by Eqn. (4.24) using only a portion of it to suppress 20 db into 1 db.

The obtained output waveforms are shown in Figures 87, 88, 89, 90 and 91.

EXAMPLE 5 - DISTORTIONLESS CHARACTERISTIC-DYNAMICS IN THE FORWARD PATH

To prove the efficiency of the derived characteristic one more example was simulated inserting dynamics in the forward path of the AGC loop.

The block diagram of the simulated loop is shown in Fig. 92.

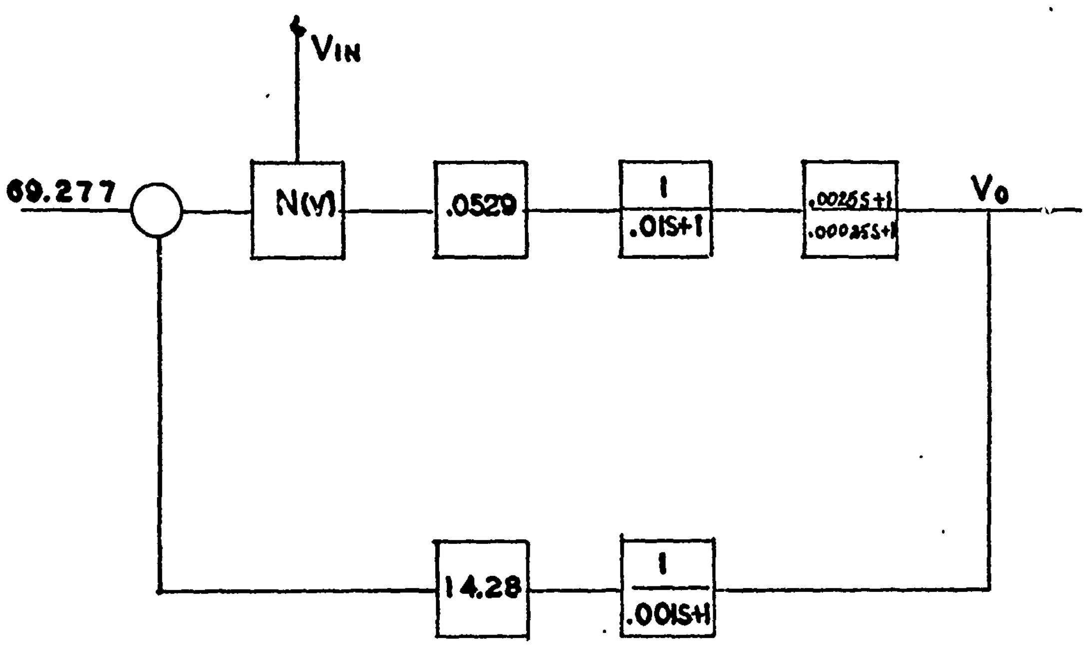


Figure 92. Block Diagram of Simulated AGC Loop.

The dynamics were given and the remaining parameters around the loop were calculated as was shown previously. As a non-linear gain characteristic the derived equation was used.

As Vin two waveforms were used, one sinusoidal and one triangular both having input amplitude varying 20 db.

A two second long output record is shown in Figures 93 and 94 for the sinusoidal and triangular waveforms correspondingly.

As it is observed from the output records the AGC action diminished the output amplitude variation to 1 db and the shape of the input waveform was preserved in an almost excellent way.

EXAMPLE 6 - DISTORTIONLESS CHARACTERISTIC-DYNAMICS IN THE FORWARD PATH FREQUENCY RESPONSE

The frequency response of the AGC loop the block diagram of which is shown in Fig. 92 was calculated using the conventional computational tool BODE (open)-NICHOLS-BODE(closed) diagrams.

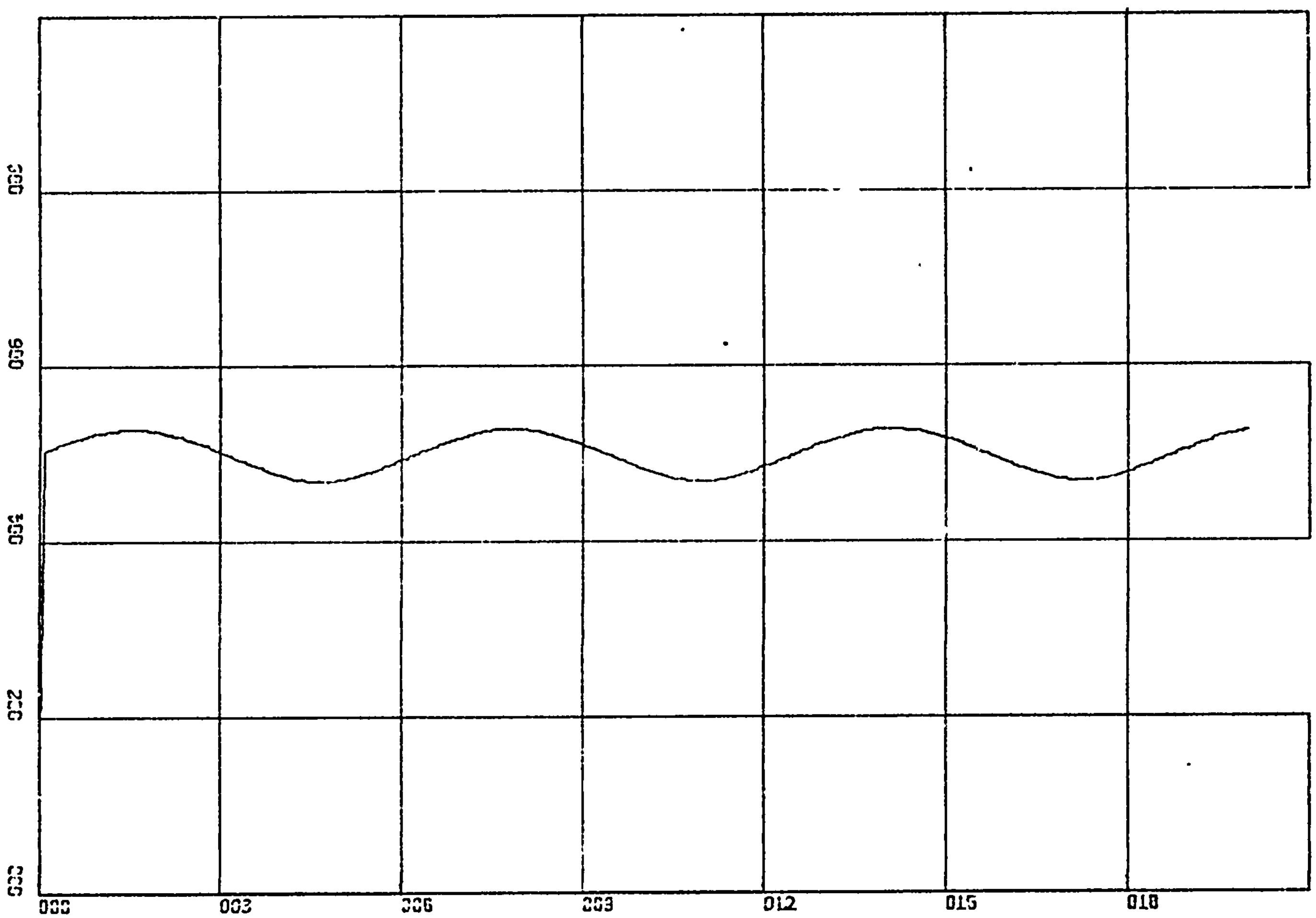
The loop gain was calculated using the formula (3-57) repeated here.

$$(\text{Loop gain}) = (\text{Linear gain}) \times (\text{Input amplitude}) \times (\text{Slope of Charact.})$$

In the present case:

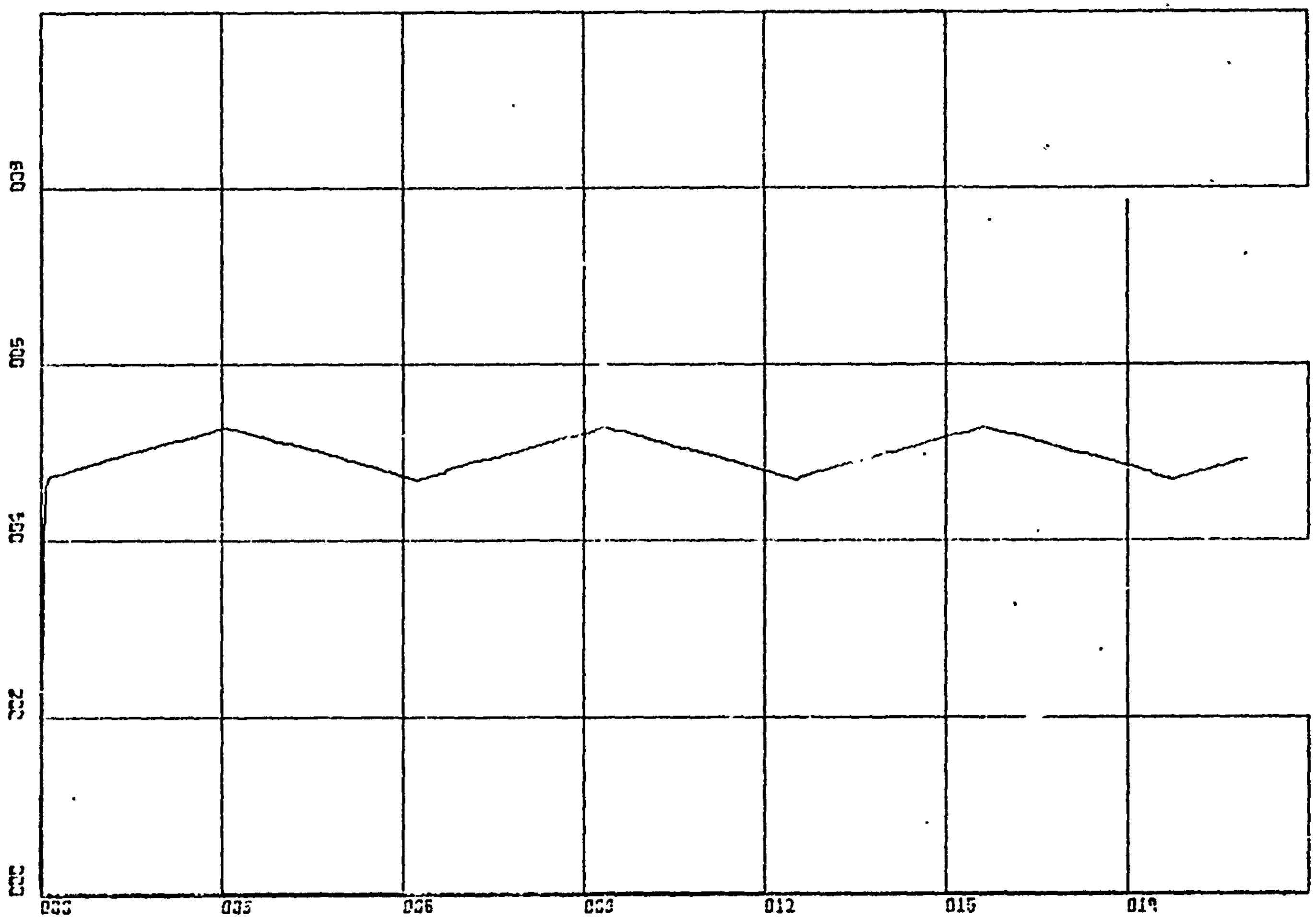
$$(\text{Linear gain}) = (.0529) \times (14.28) = .755$$

Input amplitude varies from 1 to 10 volts so taking the average is 5.5.



X: .3, Y: 2.0

Figure 93. Distortionless Characteristic, Dynamics in Forward Path, Input Waveform Sinusoidal.



X: .3, Y: 2.0

Figure 94. Distortionless Characteristic Dynamics in the Forward Path, Input Waveform Triangle.

The operating region of the non-linear gain characteristic as was calculated in the first case of example 2 of this Appendix lies between Nmax = 89.0 and Nmin = 10 with a v = 8.8756, therefore the average slope is

$$\frac{N}{v} = \frac{79}{8.87} = 8.9$$

Substituting these numbers into the above equation

(Loop gain) = (.755) x (5.5) x (8.9) = 37

which corresponds to 31.4 db.

Thus the open loop transfer function of the system becomes

$$\frac{37(.0025s+1)}{(.01s+1)(.00025s+1)(.001s+1)}$$

and the drawn BODE (open) is shown in Fig. 94.

The corresponding NICHOLS chart and BODE (closed) are shown in Figures 96 and 97.

From the last one it is observed that the AGC loop acts as a B.P. filter, as was expected for a type 0 system, with low frequency -3 db cut-off at about 1000 rad/sec and high frequency -3 db at about 40000 rad/sec.

Thus it should be expected that for frequencies outside the pass band, the AGC action takes place restricting the output amplitude extremes between the prescribed bounds and the derived non-linear characteristic takes care of the shape of the output waveform producing a distortionless output. For frequencies inside the pass-band the output should be distorted and completely unregulated. For

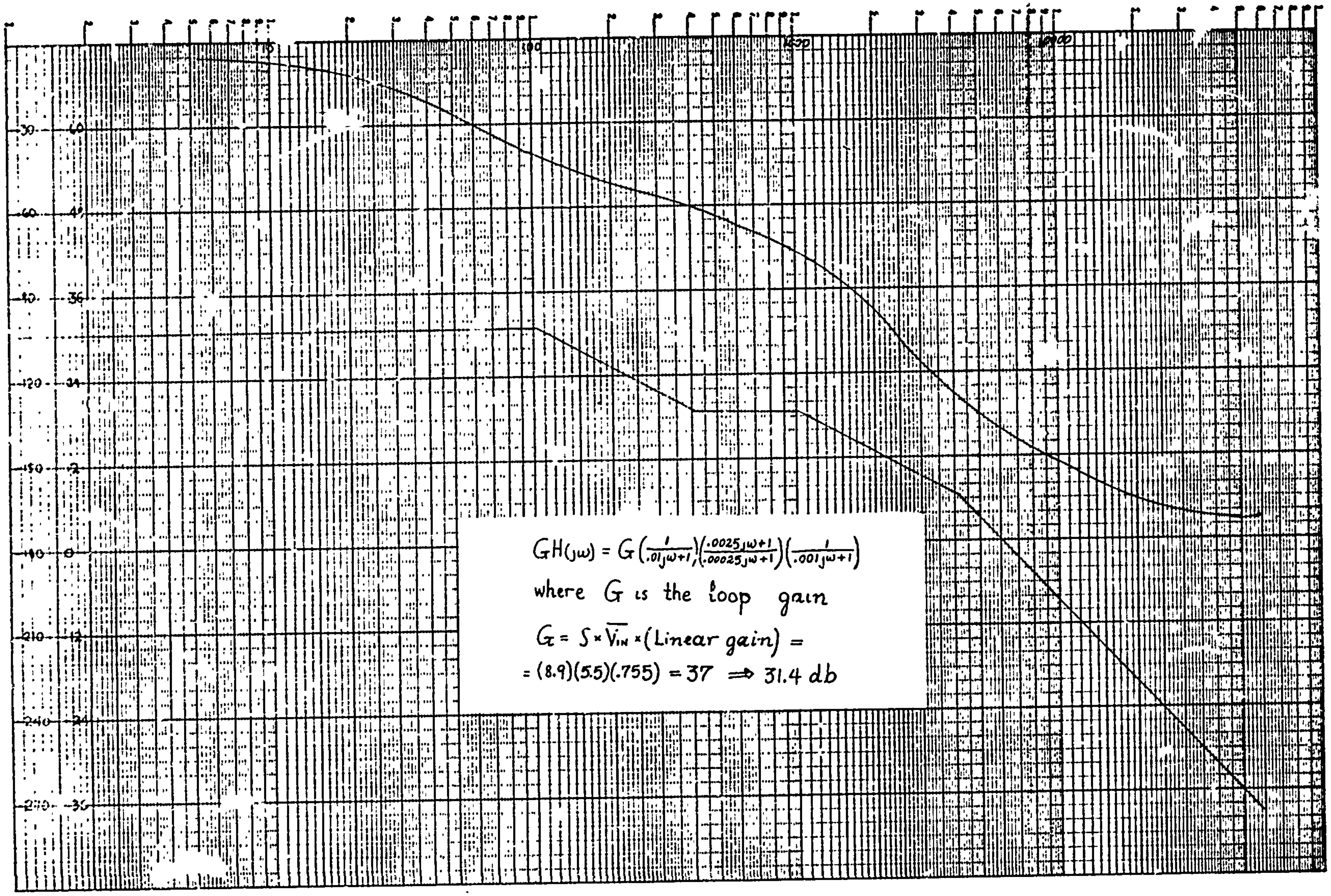


Figure 95. BODE Open of Example 6.

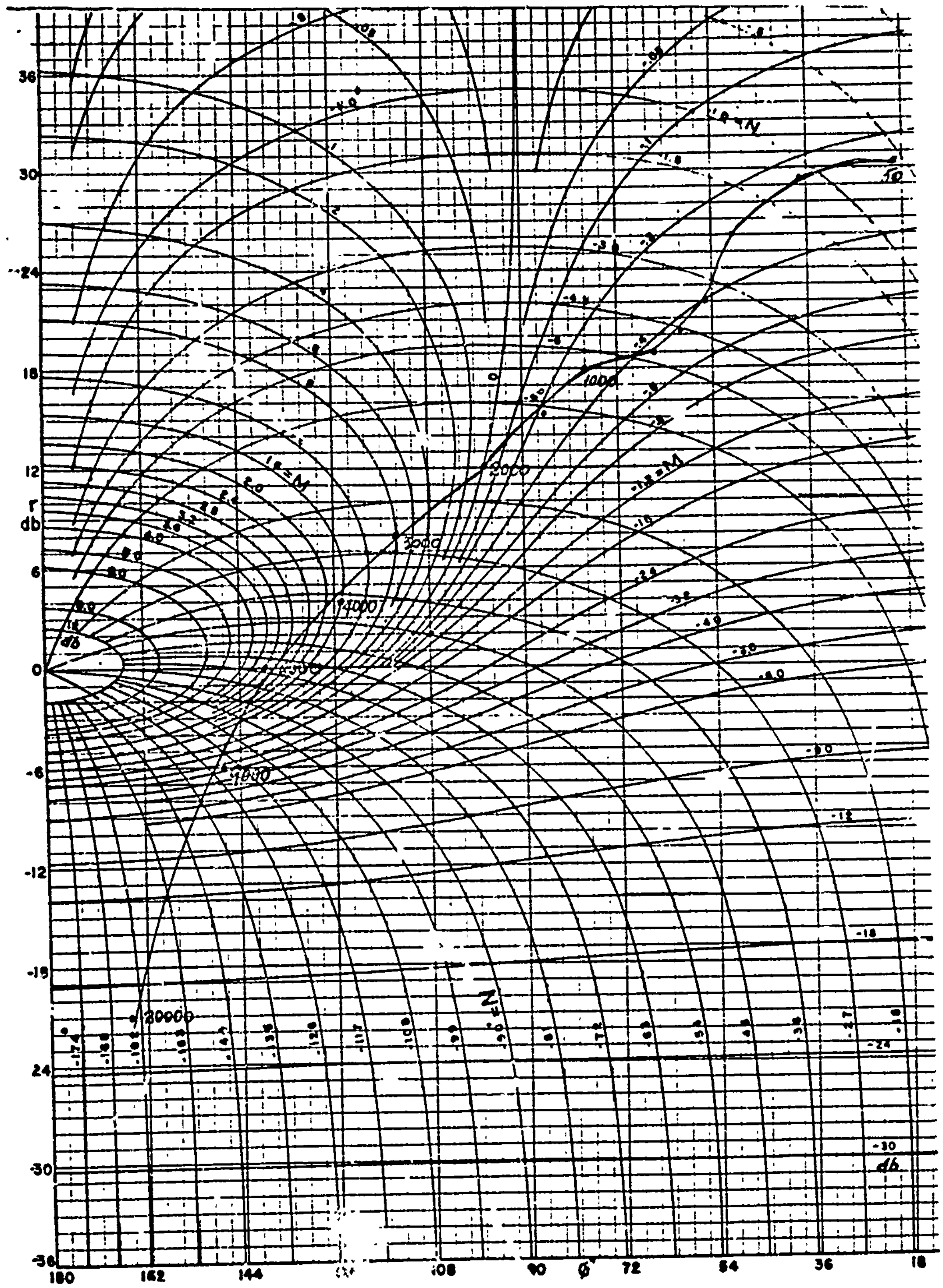


Figure 96. Nichols Plot of Example 6.

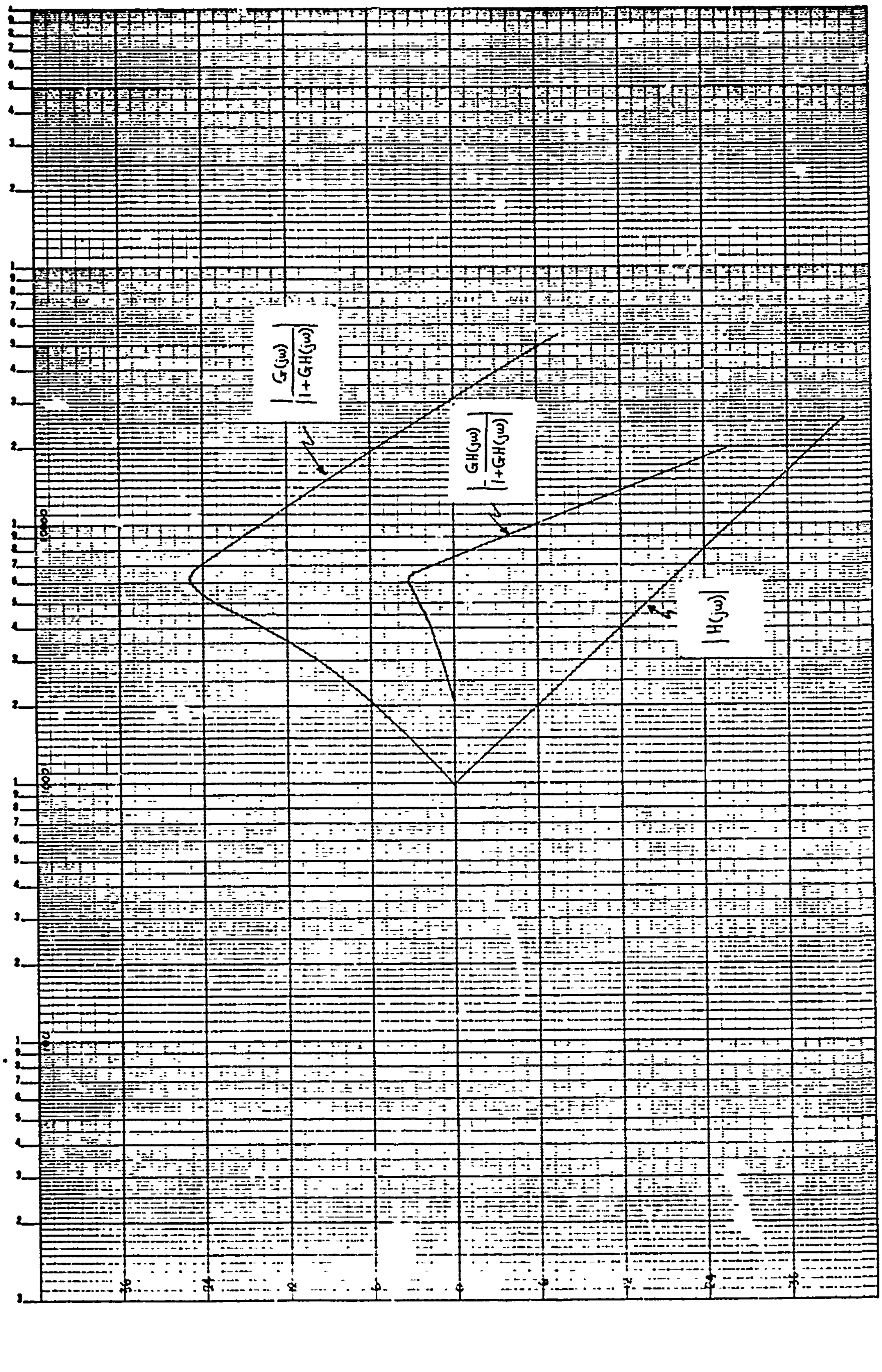


Figure 97. BODE Closed of Example 6.

intermediate cases, intermediate results between the described above extremes should be expected.

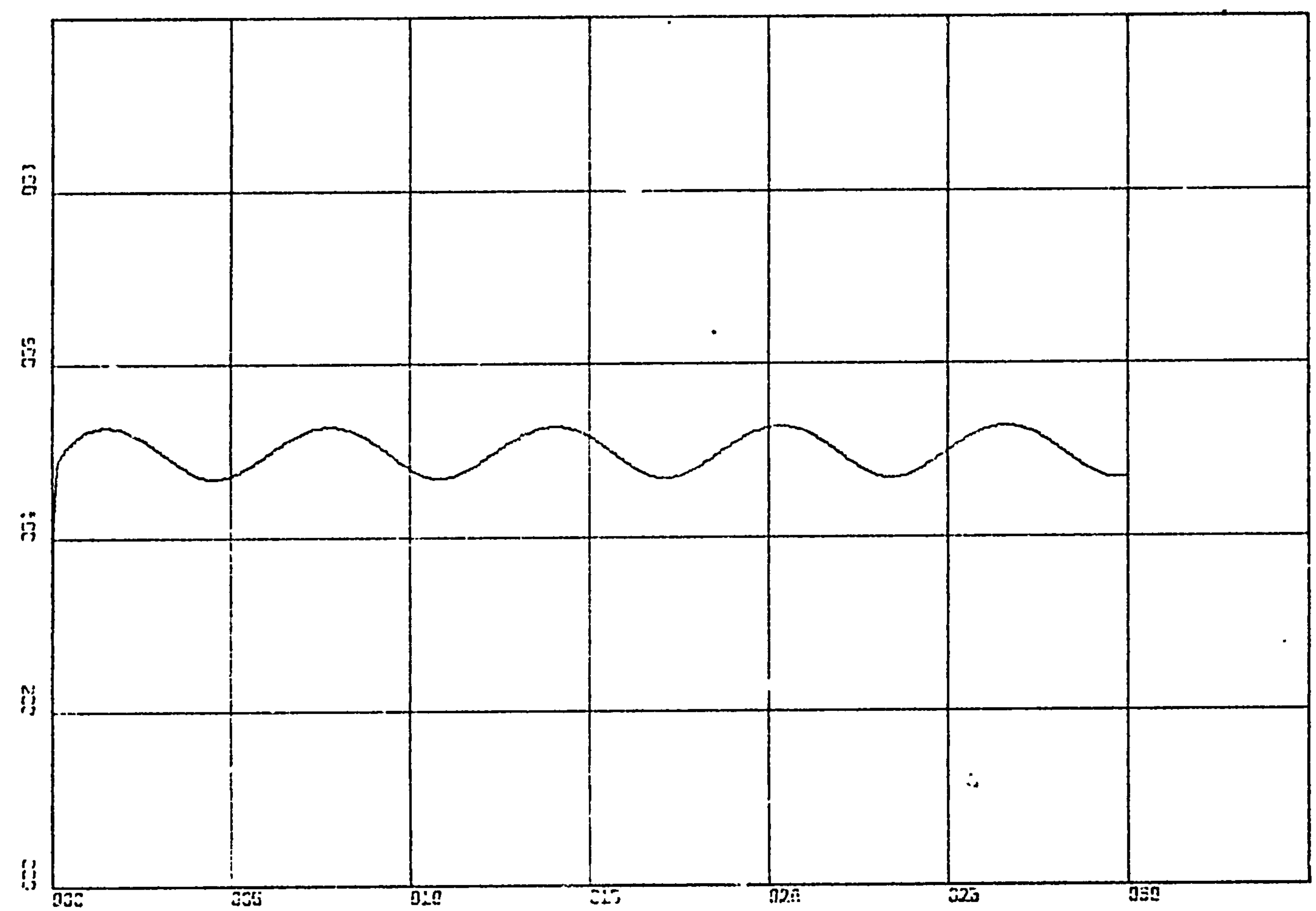
Simulation runs for input frequencies 100, 200, 400, 600, 800, 1000 and 1500 radians per second resulted in the output waveforms shown in Figures 98, 99, 100, 101, 102, 103 and 104.

In all cases the input was a sine wave with varying amplitude from 1 to 10 volts and a starting phase of 0°. The output waveforms show very clearly the above expressed ideas. Specifically it is observed that as the input frequency approaches the lower cut-off frequency, the regulation becomes poorer and the waveform shape deteriorates.

The change in phase shift through the AGC loop is also apparent as the input frequency changes.

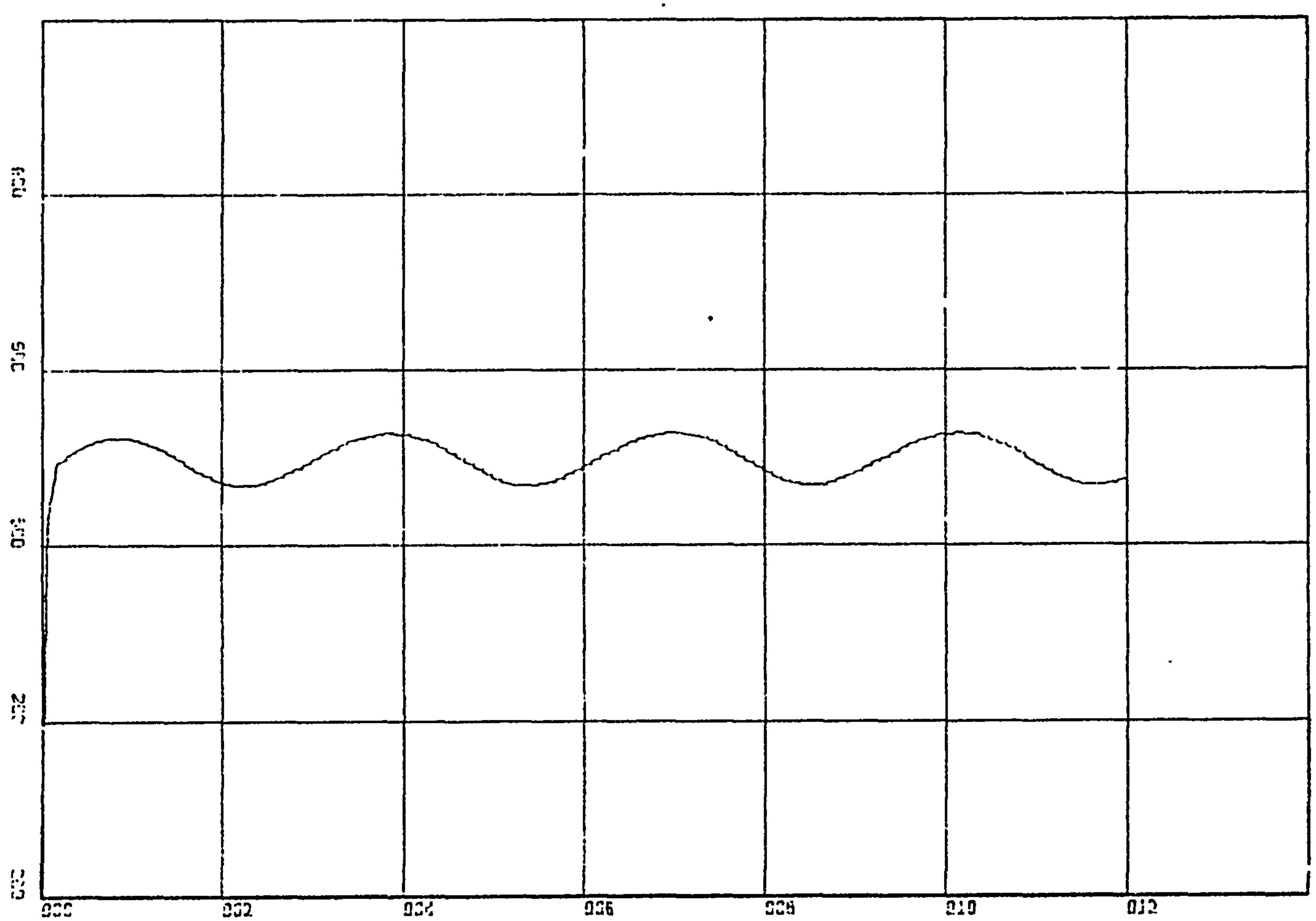
If now it is desired to work with the particular frequency ω = 1000 rad/sec the feedback pole which is at the discretion of the designer has to be chosen in such a manner that the desired frequency is outside the pass-band of the closed loop response.

For the particular case, observing that the loop gain is quite high thus the bandwidth of the system is mainly determined by the feedback pole, if this pole were chosen at ω = 2500 rad/sec it would create a low frequency -3 db cut-off point at about 2,000 rad/sec and the frequency 1000 rad/sec would be outside the closed loop pass-band.



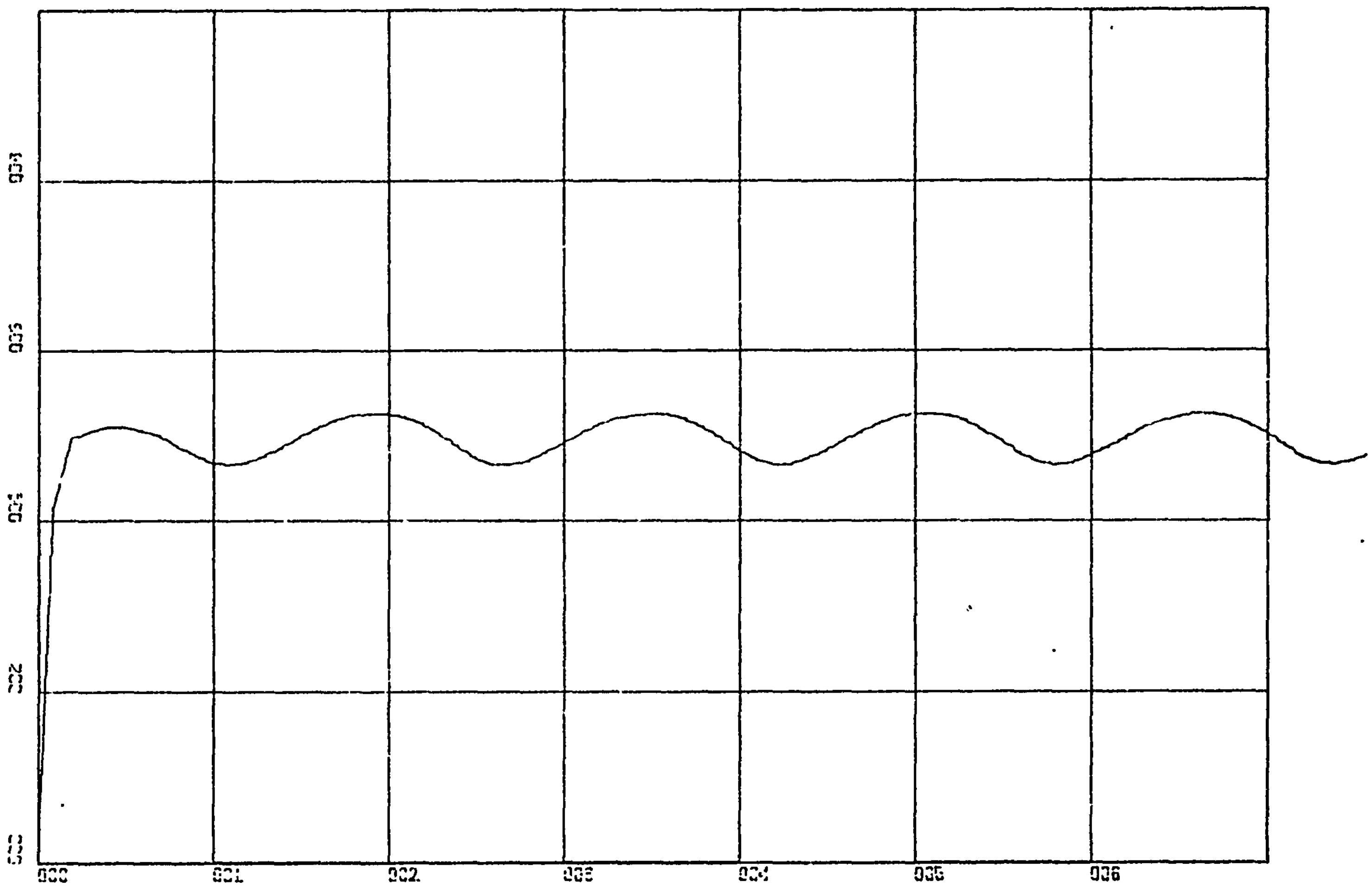
X: .05, Y: 2.0

Figure 98. Distortionless Characteristic, Dynamics in the Forward Path, Feedback Pole at 1000 rad/sec, Input Frequency 100 rad/sec.



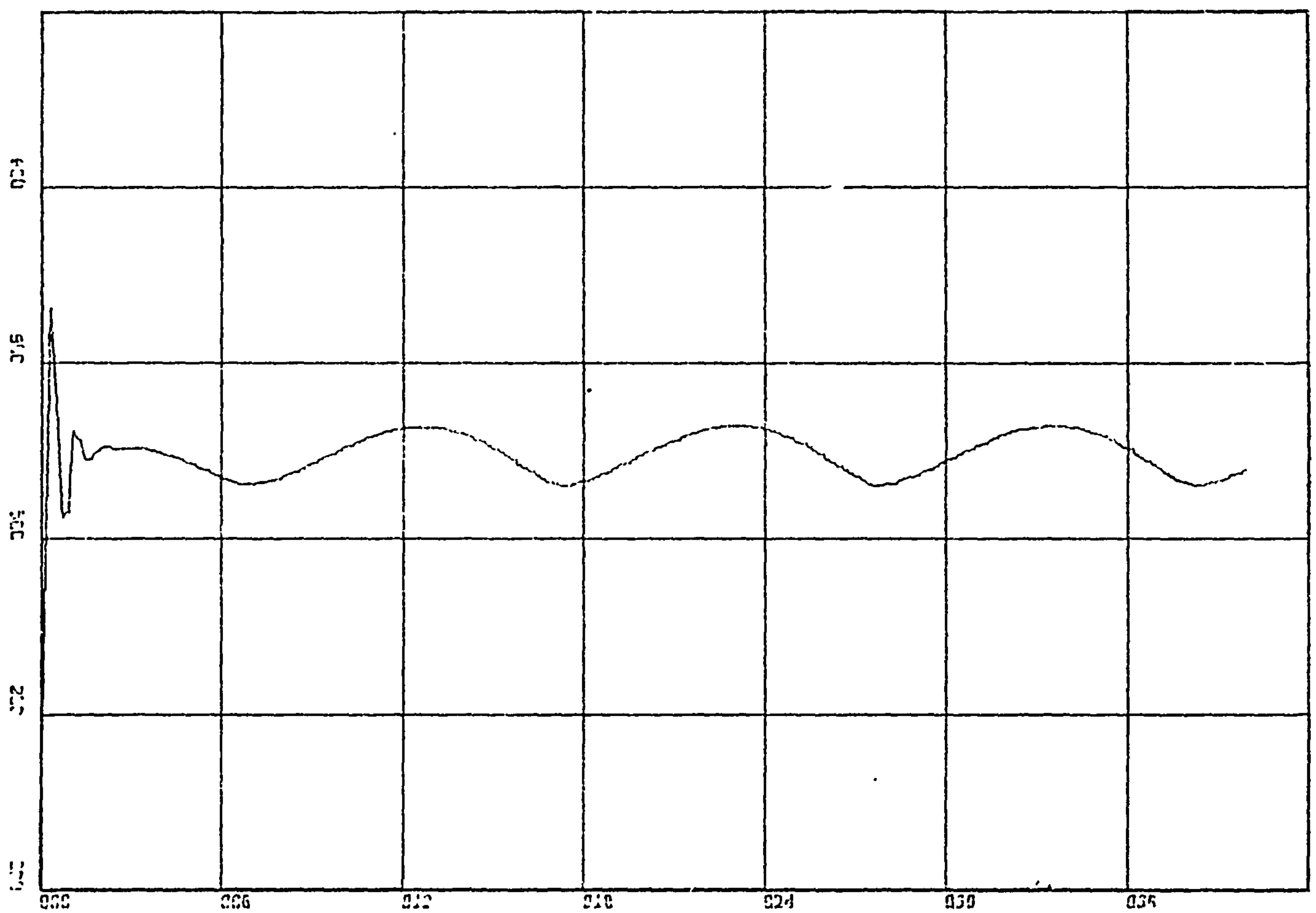
X: .02, Y: 2.0

Figure 99. Distortionless Characteristic, Dynamics in the Forward Path, Feedback Pole at 1000 rad/sec, Input Frequency 200 rad/sec.



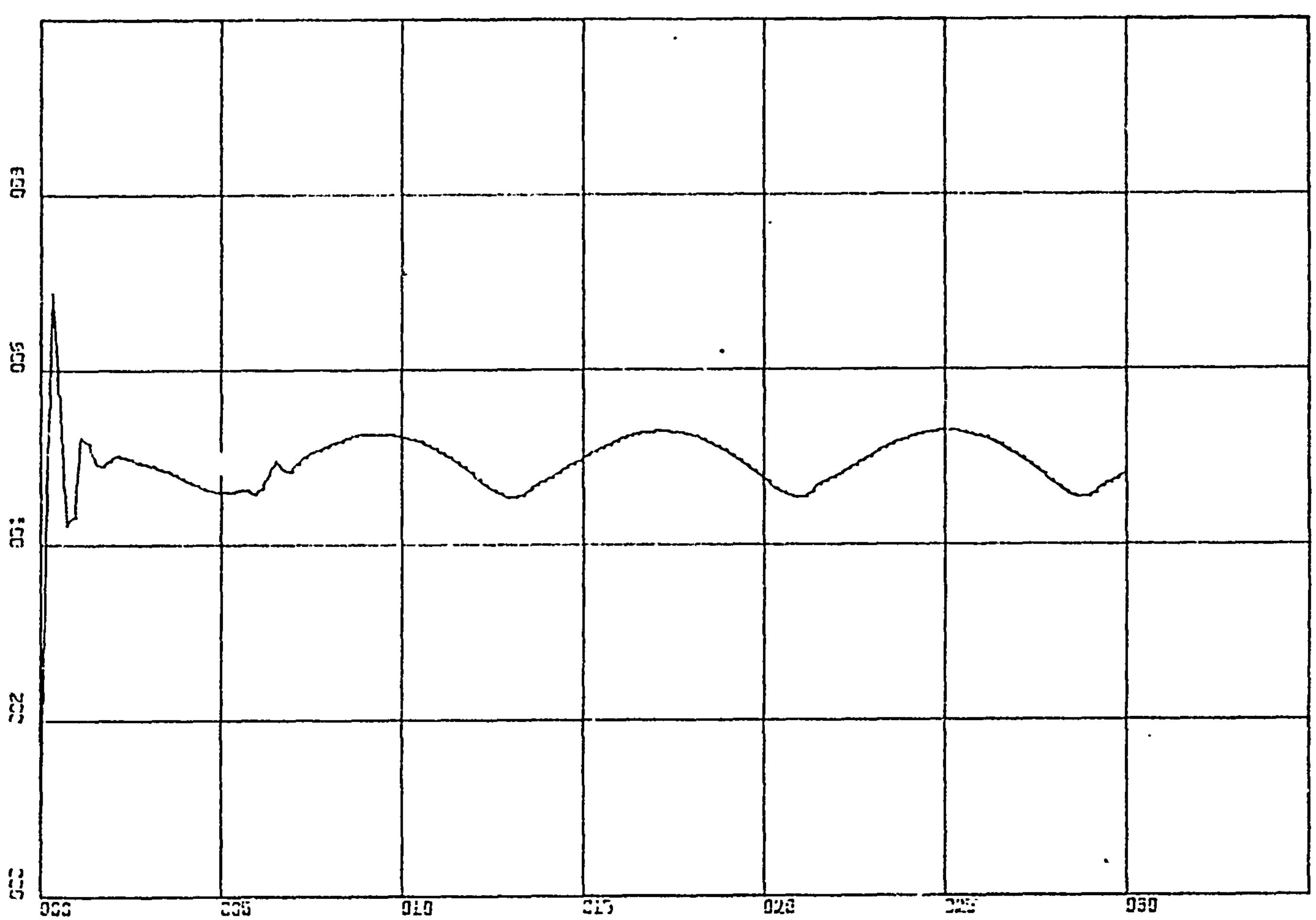
X: .01, Y: 2.0

Figure 100. Distortionless Characteristic, Dynamics in the Forward Path, Feedback Pole at 1000 rad/sec, Input Frequency 400 rad/sec.



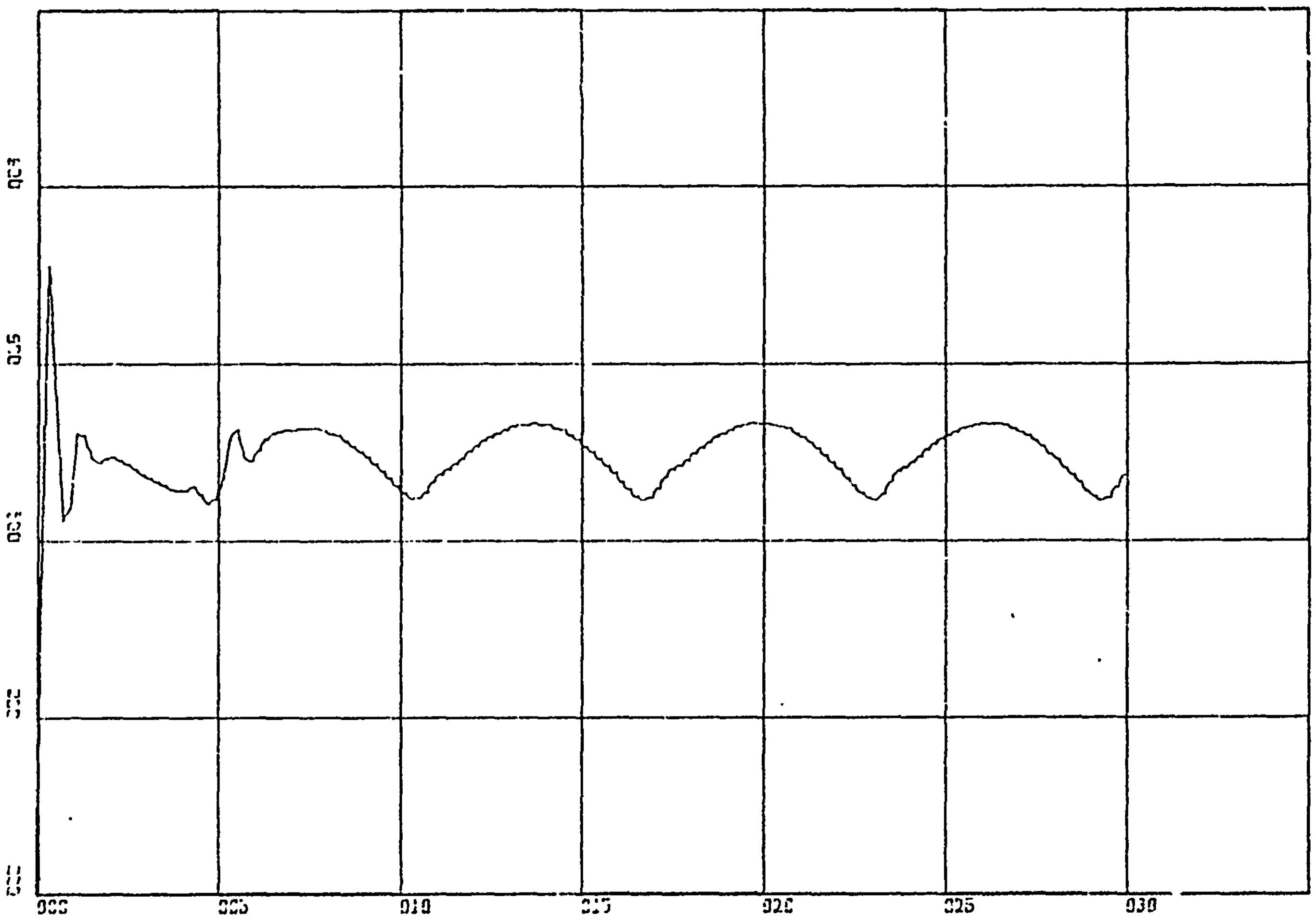
X: .006, Y: 2.0

Figure 101. Distortionless Characteristic, Dynamics in the Forward Path, Feedback Pole at 1000 rad/sec, Input Frequency 600 rad/sec.



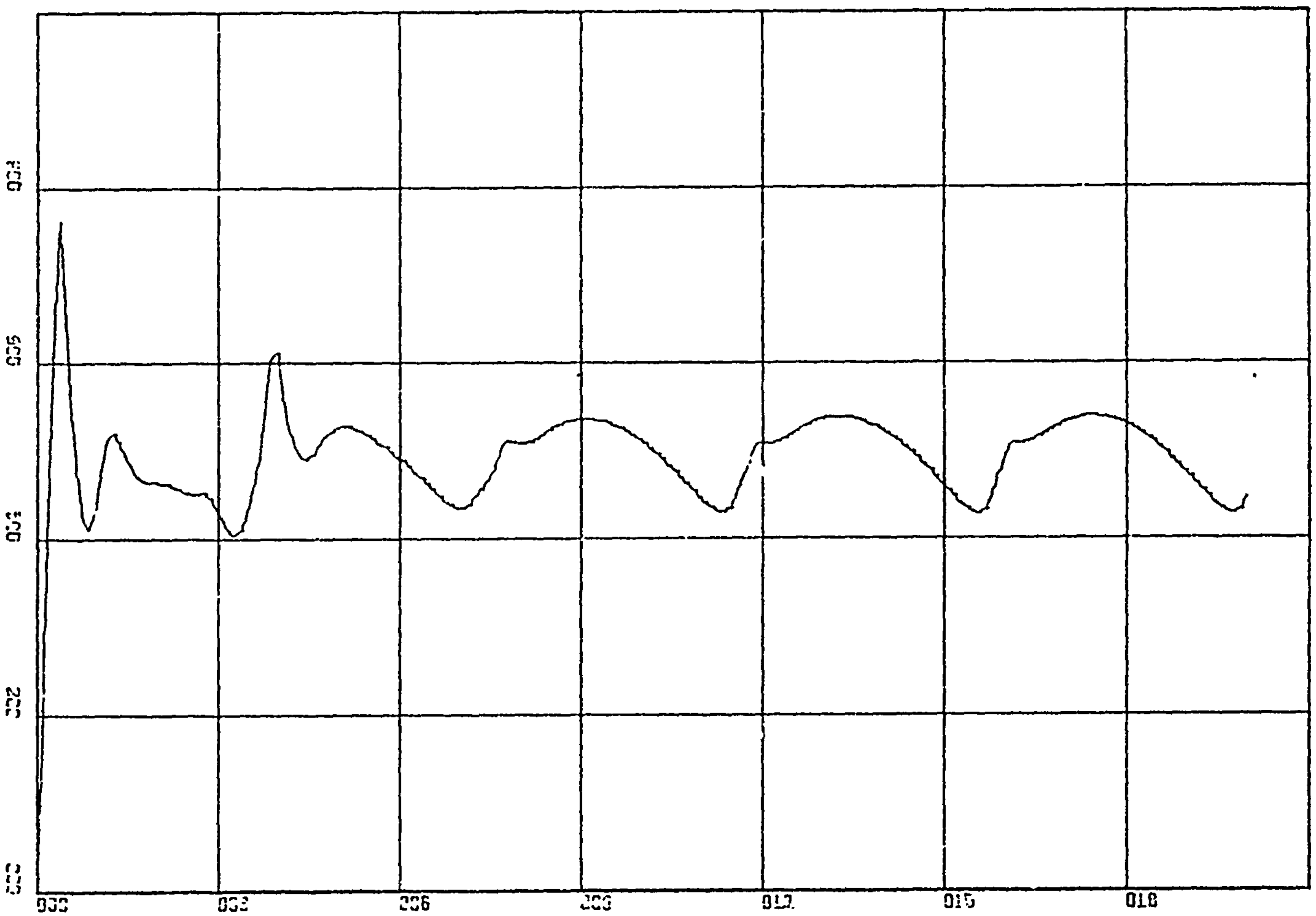
X: .005, Y: 2.0

Figure 102. Distortionless Characteristic, Dynamics in the Forward Path, Feedback Pole at 1000 rad/sec, Input Frequency 800 rad/sec.



X: .005, Y: 2.0

Figure 103. Distortionless Characteristic Dynamics in the Forward Path, Feedback Pole at 1000 rad/sec, Input Frequency 1000 rad/sec.

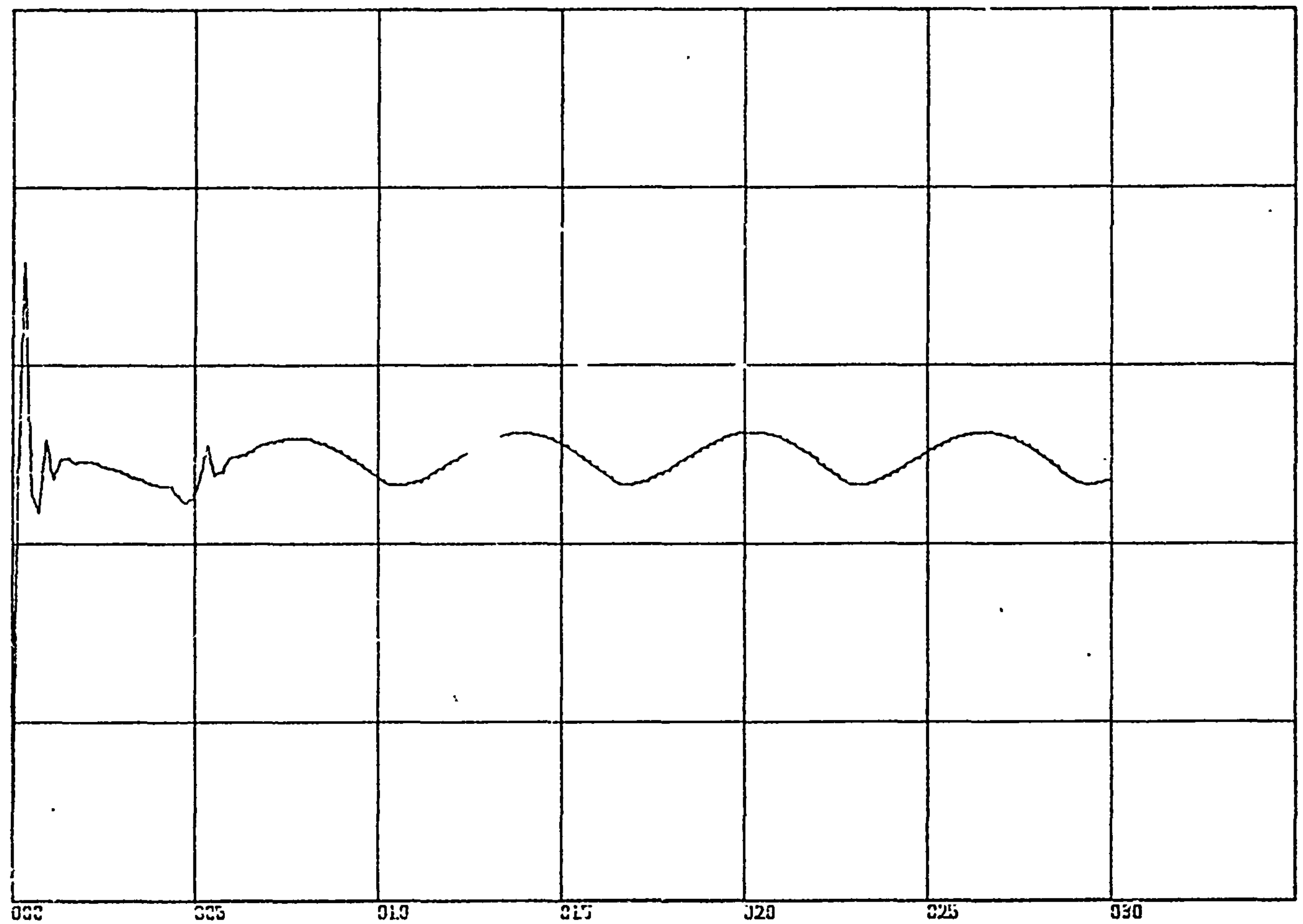


X: .003, Y: 2.0

Figure 104. Distortionless Characteristic, Dynamics in the Forward Path, Feedback Pole at 1000 rad/sec, Input Frequency 1500 rad/sec.

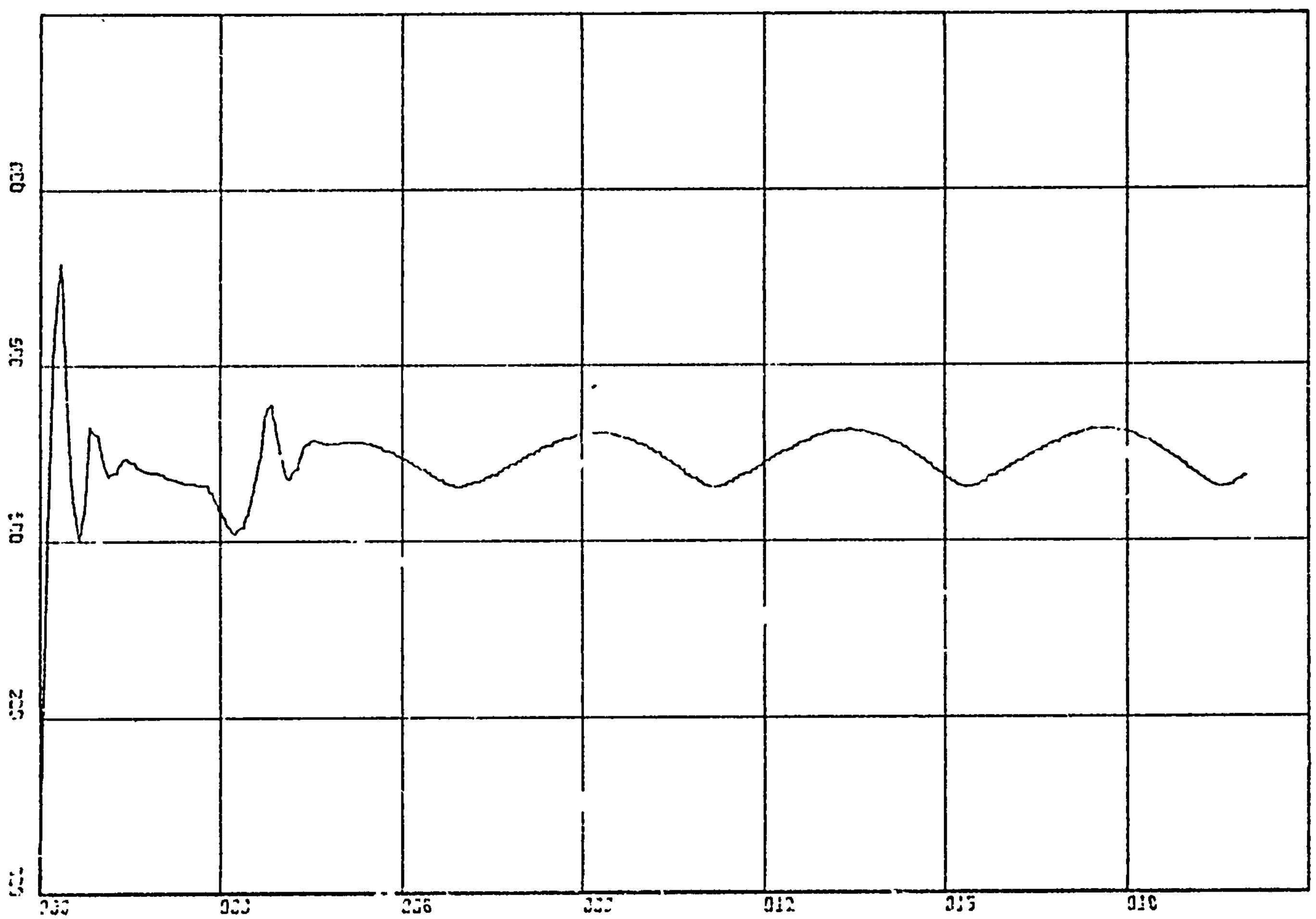
Running simulations with the new feedback pole for frequencies 1000, 1500, 2000 and 2500 rad/sec resulted in the output waveforms shown in Figures 105, 106, 107 and 108.

It is easily observed how the ideas expressed previously apply to these waveforms.



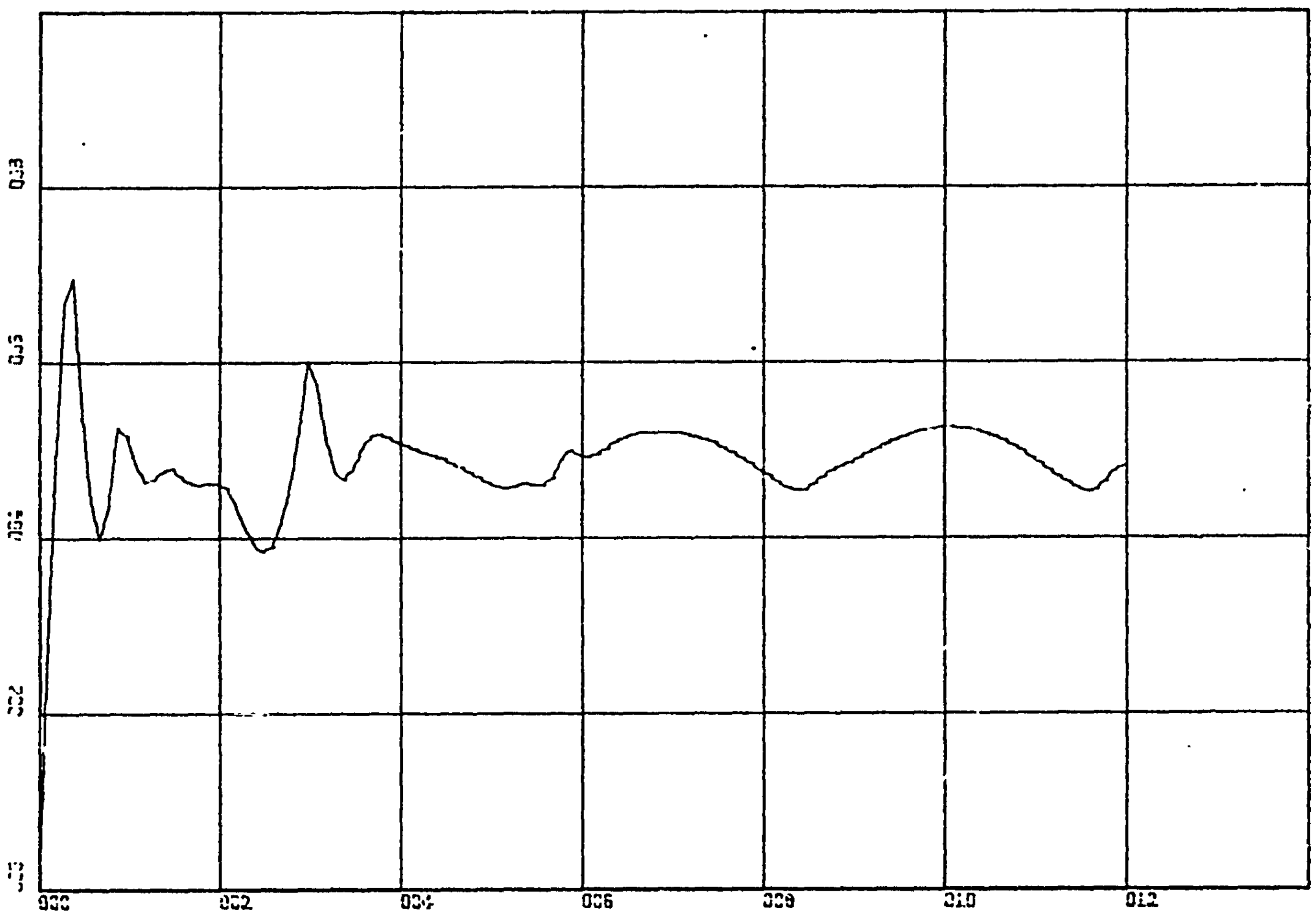
X: .005, Y: 2.0

Figure 105. Distortionless Characteristic Dynamics in the Forward Path Feedback Pole at 2500 rad/sec, Input Frequency 1000 rad/sec.



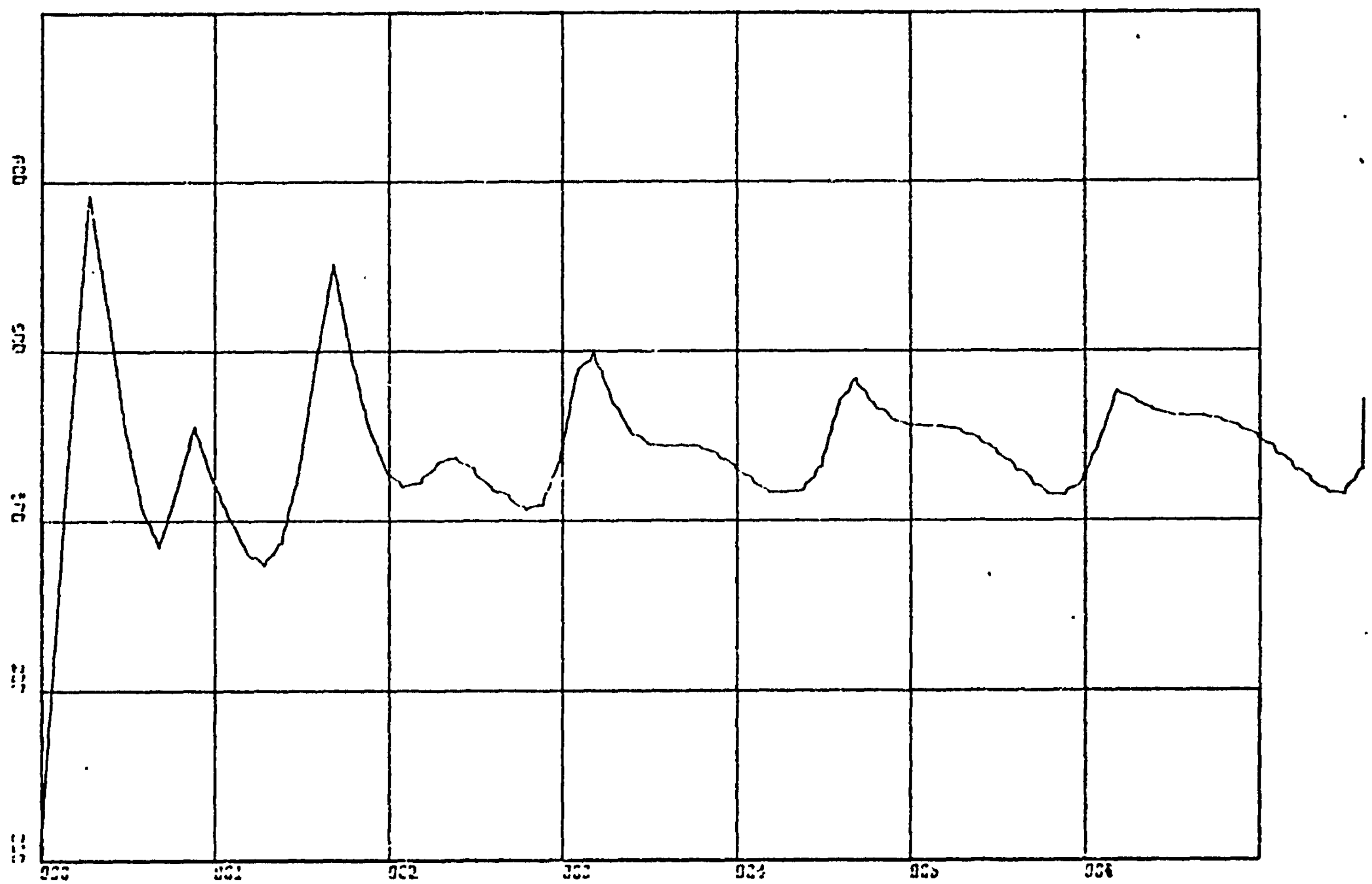
X: .003, Y: 2.0

Figure 106. Distortionless Characteristic Dynamics in the Forward Path Feedback Pole at 2500 rad/sec, Input Frequency 1500 rad/sec.



X: .002, Y: 2.0

Figure 107. Distortionless Characteristic Dynamics in the Forward Path Feedback Pole at 2500 rad/sec, Input Frequency 2000 rad/sec.



X: .001
Y: 2.0

Figure 108. Distortionless Characteristic Dynamics in the Forward Path, Feedback Pole at 2500 rad/sec, Input Frequency 4000 rad/sec.

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